The Sound of Chords

Harry Laurence Hart

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Canada

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Preface

Imagine that someone is learning to compose music. After studying some of the rudiments of music theory, fundamental questions may come to mind that can be difficult to answer. For example, what is the most appropriate chord to use at any point in time in the course of composing a specific musical composition? One approach is to apply commonly-used chord progressions in the context of a frequently-used scale. Another approach is to experiment with different chords on an instrument until the sound is deemed to be satisfactory. A third approach, used in jazz, is to replace more basic chords with various predetermined chord extensions or substitutes. A fourth approach is to consider chords that are related by minimal voiceleading.

Experience gained by composing over long periods of time can provide composers with an intuitive sense of many innovative options that may be available, leading to fresher, more nuanced, and more original compositions that better achieve the composer's artistic vision. Can some of this knowledge be captured and made available to young composers in a holistic way that includes all possible chords, including chords that some experienced composers may choose to avoid in the course of developing a personal style or may simply overlook?

In the visual arts, the colour wheel is a basic tool that allows artists to understand colours and the relation between colours at a basic sensory level. Colours are described by several variables such as their name (hue), their relative lightness and darkness (value), whether they are warm or cool, whether they can or cannot be mixed from other colours (for example, secondary and primary colours, respectively), and the effect of adding black or white. The colour wheel provides a point of departure for understanding colour in greater depth, and for making informed decisions about the use of colour.

A composer is often faced with making very abstract decisions about chord selection without the benefit of a tool like the painter's colour wheel – in other words, without the benefit of a method for accurately evaluating the sound of any chord or for comparing the sounds of different chords.

Can the most appropriate chords be determined through an understanding of the nature of the sound of chords at a basic sensory level? While it is ultimately up to the composer to determine what is appropriate, understanding the sound of chords at the sensory level is the initial step in understanding how to use chords. So, in this book, we look primarily at chords in isolation rather than chords in the context of other chords, such as chord progressions, or in the context of specific local and global pitch contexts, or in the context of tonality.

However, the graphs, charts and figures in this book provide information about how one of the illustrated chords can relate to some other chords which consist of a different numbers of pitches. With this information, the composer who has a solid understanding of the sound of chords at a basic sensory level will have a better understanding of how a single pitch, intervals, triads and trichords, selected seventh chords and tetrachords, and other selected chords with larger numbers of different pitches, relate to each other. This information opens the door to the possibility of moving back and forth between chords of different cardinality in a knowledgeable way. (The charts and graphs in this book do not show all possible chords, but the method provided can be used to create charts and graphs for any chord.)

The graphs and charts in this book also provide information that can be used by a composer to determine how well an illustrated chord would relate to prior chords in specific local and global pitch contexts at any point in time during a musical composition. Consequently, that composer will be better informed about which chords can best satisfy the composer's artistic goals at any time.

In summary, this book aims to provide young composers with a practical method for evaluating and comparing the sound of chords at the sensory level. It will also help them understand relationships between chords that contain different numbers of pitches, and help them develop chord progressions. It may also be of interest to music theorists and to neuroscientists who study the human auditory system. This book also provides a brief introduction to the foundations of harmony and tonality.

1
Constructing a Method for Evaluating and Comparing the Sound of Chords

Musicians and music theorists discuss music and music theory at many different levels of abstraction. A music teacher may ask a student to play an mf E♭ on an oboe. These instructions indicate that instrumentation, dynamics and pitch are important but the octave, in which the pitch falls, may not be. The teacher could also ask someone in an orchestra to play A_4. In this case, instrumentation and dynamics (within reason) do not matter, but pitch and octave are significant. Likewise, musicians may discuss a specific A♭ dominant seventh chord in the context of a passage in a particular musical score. In this case, pitch, letter names and accidentals, dynamics, and octaves matter. At other times, musicians will discuss dominant seventh chords in general, regardless of pitch, octave, letter names and accidentals, dynamics, or particular compositions.

Different levels of abstraction can reveal or hide information about a chord. Letter names and accidentals are useful in relating a chord to a scale, but can be less useful in relating the chord to other chords in general. On the other hand, while numbers can be very useful in describing aspects of chords in general, they may not very useful in rehearsal. A music teacher will not get far asking a student to play 6 on an oboe.

Here, the challenge is to find the most useful combination of different levels of abstraction – each one not too abstract and not too specific – that provides the most useful method for evaluating and comparing the sound of chords in general.

The ideal method would be of universal application. It would provide the same results for each chord, and comparable results for every chord, regardless whether: a chord is played by a group of trumpets or a several different woodwind instruments; a chord is played quietly or loudly; a chord is pitched in a high register or a low one; the pitches in the chord are within one octave or spread across several octaves; a chord is played with different articulations, such as pizzicato on string instruments or legato on brass instruments; and so on.

The ideal method would also be as simple and user-friendly as possible, as a colour wheel is for the visual arts.

Top-down and bottom-up approaches can both be useful in constructing a method.

1.1 The top-down approach

From a top-down perspective, notes can be represented "as points… on a circle (*pitch-class space*)", which is similar to a clock face [1]. All notes can be represented as specific pitches in pitch-class space in the form of letter names with accidentals (F♯/G♭, A♯/B♭ …), irrespective of which octaves they might occur in. Ignoring information about octaves is known as octave equivalence. This approach also ignores whether a pitch is an A♯ or a B♭, what instruments might perform the notes, and which dynamic levels and articulations might be used. One can also say that this approach includes enharmonic equivalence, instrumental equivalence, dynamic equivalence, and articulation equivalence. In pitch-class space, as on a normal clock face, the distance between each point on the circle is the same. Likewise, in an equal-tempered octave, the distance between each adjacent pitch is one semitone, which is equal to 100 "cents".

Notes can also be represented as *numbered points* on a twelve-point circle (*twelve-point circular space*), which constitutes a higher level of abstraction at which it is clear that musical notes are represented in a form in which they have ceased to be notes that are performed or that have any sound. At this level, the representation of notes as numbered points on a circle is a mathematical measure of the semitone distance between specific points. Notes are represented as numbers from 0 to 11 rather than by letter names and accidentals. At this level, notes have also ceased to have any relation to frequency (kHz). Each chord is represented by a set of numbered points that begins with the number 0. Unlike pitch-class space in which a single point represents a specific pitch (such as B♭), the number 0 in twelve-point circular space may represent any pitch and other numbered points that represent other pitches in that specific chord represent the distance in semitones from pitch number 0. By replacing letter names and accidentals with numbers, the size of our method is reduced by nearly 92% without the loss of any essential information.

Traditional set class theory (Forte set classes) provides a way of capturing and grouping all possible equal-temperament chords in a twelve-point circular space. Set class theory is described in detail in other publications, so it will not be reviewed in detail here. For now, we simply provide an example: set class 3-11 includes all major and minor triads, regardless whether they are in root position, first inversion or second inversion (as those terms are traditionally used in music theory).

For our purposes, traditional set class theory seems to be too abstract to be used as a classification method because nearly all composers and listeners distinguish between

the sound of major and minor triads. So, a slightly less abstract classification method would appear to be more appropriate.

The approach selected here is to use transpositional (Tn) classification to modify traditional set class theory: numbered-point sets that can be transformed into each other by transposition are members of one set-class. Transpositional set classes are numbered in increasing order: 2-1, 2-2, 2-3, and so on. These set classes do not include chords that are related by "inversion through reflection", which is defined for this purpose to mean a transformation in which every numbered point (x) of a numbered-point set is replaced by its inverse, calculated as 12 – x. Each Forte set class, which contains chords that are related by inversion through reflection, is split into two separate set classes, and the Forte set-class name is also split into two; the name of each of these transpositional set classes has the letter A or B added to it [2].

For example, set class 3-11 is divided into: 3-11A (which consists of the minor triad in root position, first inversion and second inversion); and 3-11B (which consists of the major triad in root position, first inversion and second inversion). Note that the term "first inversion" in this example has a meaning similar to that used traditionally in music theory to signify a chord where the traditional "root" (the lowest pitch below a stack of major and minor thirds) has been transposed up by at least one octave. Likewise, the term "second inversion" means a chord where the root and the next lowest pitch have both been transposed up by at least one octave. Here, the term "inversion" is applied to all chords and is not limited to chords constructed with a stack of major and minor thirds. Mathematically, these inversions are rotations in twelve-point circular space. In this transpositional twelve-point circular space, all semitone measurements are made in a clockwise direction. In this book, the words "inversion", "root position" ,"first inversion" and "second inversion" refer to rotations in twelve-point circular space; all references to "inversion through reflection" specifically use that term.

The transpositional set classes are set out in the form "TSC*-*" (for example, TSC2-1, TSC2-2, TSC2-3, TSC3-11A, and TSC3-11B).

As an example of this transpositional twelve-point circular space, every major triad in root position is abstracted to the same numbered point set (0-4-7).

In Figure 1.1 below, major triads in root position (0-4-7) are represented in transpositional twelve-point circular space. Rotating the dot pattern counter-clockwise, so that pitch 4 becomes pitch 0, shows the first inversion major triads (0-3-8). Rotating the pattern counter-clockwise again, so that pitch 7 becomes pitch 0, produces the second inversion major triads (0-5-9). These three numbered-point sets represent the three chords contained in the transpositional set class TSC3-11B. In twelve-point circular space, the number 0, when used in different sets of numbered points to represent the inversion of chords within one transpositional set class, represents a

different pitch in each inversion. For example, (0-4-7), (0-3-8) and (0-5-9) may represent (C-E-G), (E-G-C) and (G-C-E). Depending on the inversion, the numbered point 0 would represent C, E or G.

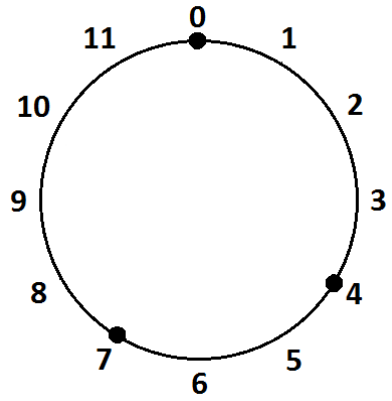


Figure 1.1 Major triads in root position (0-4-7).

In Figure 1.2 below, major triads in root position (0-4-7) are represented in transpositional twelve-point circular space as solid black dots. Numbered points 4 and 7 are reflected through the 0-6 axis, (numbered points 8 and 5 are represented as small circles), producing the second inversion minor triads (0-5-8). This is an example of inversion through reflection through the 0-6 axis.

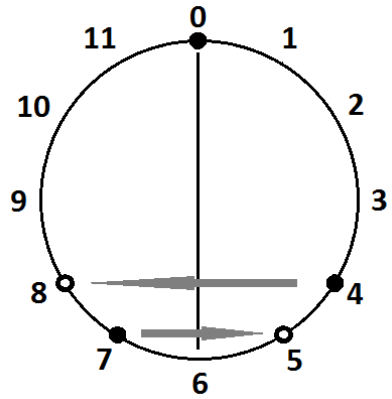


Figure 1.2 Inversion through reflection of major triads in root position (0-4-7) into second inversion minor triads (0-5-8).

In Figure 1.3 below, second inversion minor triads (0-5-8) are represented in transpositional twelve-point circular space. Rotating the dot pattern counter-clockwise, so that numbered point 5 becomes numbered point 0, shows the root position minor triads (0-3-7). Rotating the pattern counter-clockwise again, so that numbered point 8 becomes numbered point 0, produces the first inversion minor triads (0-4-9). These three numbered-point sets represent the three chords contained in the transpositional set class TSC3-11A.

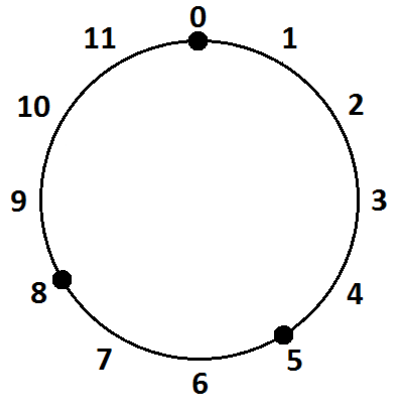


Figure 1.3 Second inversion minor triads (0-5-8).

So far, the top-down approach has provided us with a framework that can accommodate all equal-tempered chords in a transpositional twelve-point circular space. Chords are represented by the distance between numbered points, measured in semitones in a clockwise direction.

The classification of chords as transpositional set classes results in one single pitch represented in 1 TSC1 set class, 11 intervals in 6 TSC2 set classes, 55 triads and trichords in 19 TSC3 set classes, and 165 seventh chords and tetrachords in 43 TSC4 set classes.

However, this framework is completely free of sound. Musically, it is completely sterile.

It is clear that a different approach is needed to take us further. It is necessary to consider how chords are actually performed and perceived.

1.2 The bottom-up approach

From a bottom-up perspective, our approach draws on research that has been carried out by Ernst Terhardt, Dale Purves, and others.

The relationship between the sound of the human voice and the sound of musical instruments, such as strings and woodwinds, is essential to understanding the sound of chords made by singers and by instruments that produce vibrating strings or columns of air.

The following quote from Daniel Bowling and Dale Purves indicates the essential relationship between the sound of musical tones and chords, human vocalization, and the harmonic series.

> "Most natural sounds, such as those generated by forces like wind, moving water, or the movements of predators or prey, have little or no periodicity. When periodic sounds do occur in nature, they are almost always sound signals produced by animals for social communication. Although many periodic animal sounds occur in the human auditory environment, the most biologically important for our species are those produced by other humans […].
>
> Like most musical tones, vocalizations are harmonic. As with strings, vocal fold vibration generates sounds with a fundamental frequency and harmonic overtones at integer multiples of the fundamental. The presence of a harmonic series is thus characteristic of the sound signals that define human social life, attracting attention and processing by neural circuitry that responds with special efficiency. With respect to music, these facts suggest that our attraction to harmonic tones and tone combinations derives in part from their relative similarity to human vocalization. To the extent that our appreciation of tonal sounds has been shaped by the benefits of responding to conspecific vocalization, it follows that the more voice-like a tone combination is, the more we should "like" it." [3]

These relationships are integral to our approach to constructing a method for evaluating and comparing the sound of chords.

So, from a bottom-up perspective, we start with chords as they are actually performed and perceived and then abstract away as much unessential information as possible [4]. Accordingly, we modify the framework developed using the top-down approach.

Among the octaves within which humans normally speak and sing and most instrumental pitches are performed, we recognize the distinctive sound of a major triad in root position regardless of the octave or octaves in which the pitches are performed. Likewise, we recognize that sound regardless of the dynamic levels and articulations used in the performance. Using the terminology of psychoacoustics, we also recognize that distinctive sound regardless of sound pressure levels, critical bands, roughness, and pitch shifts. So, all of these variables can be safely discarded.

We also recognize the distinctive sound of a major triad in root position regardless of instrumentation or timbre. We recognize that sound whether it is played on a piano; played by a trio of string, woodwind or brass instruments; or sung by tenor, alto and soprano voices. This degree of universality suggests that voice, instrumentation and timbre can also be discarded. In all cases, some elements of timbre can be safely discarded, such as the "envelope" (for example, onset time, attack, sustain, spectral evolution, and decay). However, as we discuss further below, there is an inherent need to retain some information about voice, instrumentation and timbre (such as the frequency and amplitude of the harmonics of each performed note).

As it turns out, for our first dimension of the sound of chords – harmonic similarity of a performed chord – we can discard specific data about voice, instrumentation and timbre.

1.2.1 The harmonic similarity of a performed chord

Kamraan Gill and Dale Purves sought to determine whether the scales preferred in music worldwide and throughout history had an overall similarity to the spectral characteristics of a harmonic series [5]. They noted that, although humans can distinguish about 240 different pitches over an octave in the mid-range of hearing, they limited potential scale tones to 60 pitches to reduce the computational burden. They analysed 455,126 possible pentatonic scales, 45,057,474 possible heptatonic scales and a random sample of 10 million possible dodecatonic scales. Their research showed that the most widely used scales were comprised of intervals with the greatest overall spectral similarity to the harmonic series.

The following quote from the research of Gill and Purves indicates the relationship between human speech and the frequencies of harmonics in the harmonic series. In this approach, timbre (which includes the relative amplitudes of harmonics) is expressly ignored. Implicitly, octave equivalence and dynamic equivalence apply within the normal range of human speech.

> "[…] we favor a biologically based preference for harmonic series as the most plausible explanation for the particular scales used to make music over history and across cultures.
>
> Like any other sensory quality, the human ability to perceive tonal (i.e., periodically repeating) sound stimuli has presumably evolved because of its biological utility. In nature, such sound stimuli typically occur as harmonic series produced by objects that resonate when acted on by a force. Such resonances occur when, for example, wind or water forces air through a blowhole or some other accidental configuration, but are most commonly produced by animal species that have evolved to produce periodic sounds for social communication and ultimately reproductive success (e.g., the sounds of stridulating insects, the vibrations produced by the songbird syrinx, and the vocalizations of many mammals). Although all these harmonic stimuli are present in the human auditory environment, the vocalizations of other humans are presumably the most biologically relevant and frequently experienced.
>
> In humans, vocal stimuli arise in a variety of complex ways, not all of which are harmonic. Harmonic series depend on vocal fold vibrations and

> are characteristic of the ''voiced speech'' responsible for vowel sounds and some consonants. Although the relative amplitudes of harmonics are altered by filtering effects of the supralaryngeal vocal tract resonances to produce different vowel phones, the frequencies of harmonics remain unchanged. In consequence, the presence of a harmonic series is a salient feature of human vocalizations and essential to human speech and language. It follows that the similarity of musical intervals to harmonic series provides a plausible biological basis for the worldwide human preference for a relatively small number of musical scales defined by their overall similarity to a harmonic series." [6]

Purves, together with Deborah Ross and Jonathan Choi, indicated in another article that formant ratios in human speech are more closely related to just intonation than to equal temperament.

> "Ten of the 12 intervals generated by the analysis of either English or Mandarin vowel spectra are those used in just intonation, whereas 4 of the 12 match the Pythagorean tuning and only 1 of the 12 intervals matches those used in equal temperament." [7].

The methodology used by Gill and Purves is best described in their own words below. It is clear from their description that frequency ratios are a critical aspect of their method, which encompasses the frequencies of fundamental pitches in a scale as well as their spectral pitches and virtual pitches. They also consider each scale in isolation, unfettered by the concept of tonality, which would favour certain intervals over others; all intervals are treated equally.

> "The […] approach we take here is to quantitatively compare the harmonic structure that defines each interval in a possible scale to a harmonic series […] Accordingly our analysis does not depend on intervals and scales precisely mimicking a harmonic series, but evaluates degrees of similarity. The average similarity of all intervals in the scale is then used as a measure of the overall similarity of the scale under consideration to a harmonic series." [8]

> "Perceptually, the greatest common divisor of the dyad corresponds to its virtual pitch (or missing fundamental) and is used in much the same way as in algorithms that determine virtual pitch. Since the robustness of a virtual pitch depends on how many of the lower harmonics are present in the stimulus, this measure of similarity is both physically and perceptually relevant. For example, a dyad whose spectrum comprises 50% of the harmonic frequencies in a harmonic series would evoke a stronger virtual pitch perception than a dyad with only 10% of these frequencies. We refer to this metric as the percentage similarity of a dyad.

> Percentage similarity can be expressed as ((x+y-1)/(x*y))*100, where x is the numerator of the frequency ratio and y is the denominator of the ratio. For instance, a major third has a frequency ratio of 5:4; since x= 5 and y = 4, the percentage similarity is 40%.
>
> The overall conformance of a scale to a harmonic series was then determined by calculating the mean percentage similarity of the dyads in the scale in question. Using the mean as an index of similarity between a scale and a harmonic series implies that all possible dyads in the scale are equally relevant. Although in contemporary Western music any two notes in a scale can, in principle, be used together in melody or harmony, in traditional Western voice-leading and in other musical systems (e.g., classical Indian) particular tone combinations are avoided or prohibited. Nonetheless, there is no universal rule that describes which intervals might be more important in a scale than others; thus we treated all intervals equally." [9]

Gill and Purves used justly-tuned intervals in their methodology [10]. However, our top-down approach led us to a framework that accommodates all equal-tempered chords in a transpositional twelve-point circular space. Of course, music may be performed with equal-tempered intonation or just intonation. Should we modify our approach?

As Alfred Blatter notes:

> "The obvious characteristic that one hears on an even-tempered keyboard is that all perfect fifths and perfect fourths are flawed, and the thirds are of the incorrect size. […] No professional vocal or instrumental ensemble would tolerate such intonation, but the keyboardist has no choice. […] When music is performed by voices, or on nonkeyboard instruments, semitones are not all the same size. Nor are all whole tones the same. […] During performances, tuning adjustments are constantly being made." [11]

Gill and Purves observed that:

> "The use of justly tuned intervals Western music over the last few centuries has been based on equal temperament tuning, which developed as a compromise between the aesthetic value of maintaining justly tuned intervals (i.e., intervals defined by relatively small integer ratios) and the practical need to facilitate musical composition and performance in multiple keys, especially on keyboard instruments. Just intonation is generally considered the most natural tuning system […]. Moreover, just intonation is used in non-Western traditions such as classical Indian music." [12]

It is necessary to choose between equal temperament tuning and just intonation. We consider the observations above to be sufficiently compelling to justify the use of just intonation rather than equal temperament in our calculation of the harmonic similarity values of chords.

We adopt, with minor modification, the method used by Gill and Purves for analyzing scales to produce our first dimension of the sound of chords: the harmonic similarity of performed chords to the harmonic series produced by human voices and musical instruments that cause the vibration of strings or columns of air.

In a way that is very similar to the methodology used by Gill and Purves but slightly simplified, the harmonic similarity of chords in this book is analyzed with a collection of justly-tuned intervals. However, each interval has only one percentage similarity value (Figure 1.4) [13]. For example, the harmonic similarity of every major second interval that appears anywhere in a chord is calculated using the frequency ratio 9:8 and hsim 22.22, and the more remote frequency ratio 10:9 and hsim 20.00 are not used.

Transpositional twelve-point circular space with equal temperament	Equal-tempered intervals	Equal-tempered interval size (cents)	Transpositional twelve-point circular space with just intonation	Just intervals	Just interval size (cents)	Hsim of just intervals
0			0			
1	0-1	100.00	1.1944	0-1.1944	119.44	13.33
2	0-2	200.00	2.0391	0-2.0391	203.91	22.22
3	0-3	300.00	3.1564	0-3.1564	315.64	33.33
4	0-4	400.00	3.8631	0-3.8631	386.31	40.00
5	0-5	500.00	4.9804	0-4.9804	498.04	50.00
6	0-6	600.00	5.8251	0-5.8251	582.51	31.43
7	0-7	700.00	7.0196	0-7.0196	701.96	66.67
8	0-8	800.00	8.1369	0-8.1369	813.69	30.00
9	0-9	900.00	8.8436	0-8.8436	884.36	46.67
10	0-10	1000.00	10.1760	0-10.1760	1017.60	28.89
11	0-11	1100.00	10.8827	0-10.8827	1088.27	18.33

Figure 1.4 Transpositional twelve-point circular space, intervals, interval sizes, and the harmonic similarity of just intervals.

The formula for calculating the harmonic similarity (hsim) of chords, as used in this book, is set out in Appendix B (How harmonic similarity (hsim) values were calculated).

For convenience, we express the transpositional twelve-point circular space in equal-temperament form rather than just-interval form (0, 1, 2... rather than 0, 1.1944,

2.0391…). This is essentially no different than notating pitches on equally-spaced musical staves, centred on lines or between lines, with no adjustment for just intonation. Likewise, we express intervals and chords in equal-temperament form rather than justly-tuned form (a perfect fourth is 0-5 rather than 0-4.9804). But all of our calculations of harmonic similarity are based on our simplified model of just intonation.

We also treat all numbered point sets as chords, including sets that might also be used to represent scales. In other words, there is no octave doubling of any numbered points to represent the octave doubling of a tonic (or any other pitch).

For the purposes of this dimension of the sound of chords, some aspects of the framework created using the top-down approach are retained, and others are modified.

The modified framework retains all transpositional set classes. All notes continue to be represented as numbers from 0 to 11 rather than by letter names and accidentals, similar to a normal clock face. Each chord is represented by a numbered point set that begins with the number 0. Chords are represented by the distance between numbered points, measured in semitones in a clockwise direction. Individual notes do not have any relation to any specific frequencies (kHz). This framework retains octave equivalence, enharmonic equivalence, instrumental equivalence, dynamic equivalence, and articulation equivalence. It implicitly assumes that all pitches are equally and completely clear. The framework also continues to disregard the following tacitly ignored psychoacoustic concepts: sound pressure levels, critical bands, roughness, the dominant spectral region, and pitch shifts. Likewise, timbre (which includes the relative amplitudes of harmonics) is ignored.

Several aspects of the framework have been modified. The framework now uses just intonation rather than equal-temperament. All calculations of harmonic similarity are based on our simplified model of just intonation. Frequency ratios, which are a vital characteristic of this dimension, are captured in the just intervals used to calculate harmonic similarity.

Nonetheless, intervals and chords are expressed in equal-temperament form rather than justly-tuned form. Although the distance between each adjacent numbered point on the transpositional twelve-point circular space may appear to be identical, it is not. We do not adjust the distance between numbered points to account for just intonation, just as the positioning of pitches on musical staves are not adjusted for just intonation.

These modifications turn our highly-abstract top-down framework into a more musically meaningful tool that takes into account various aspects of human perception that are significant at a basic sensory level.

1.2.2 Spectral pitches and virtual root candidates

The work of Ernst Terhardt is vital to understanding the sound of chords. Research by Terhardt and his colleagues about the use of Fourier analysis and subharmonic matching leads to considerable insight into how the auditory processing system in the human brain creates the perception of chords, which is a significant advance over discussions of chordal pitches and the nature of the harmonic series, which is where many analyses end.

When a chord is performed, humans perceive the fundamental frequency of each performed note as well as spectral pitches that result from their harmonic overtones. The auditory processing system in the human brain also infers virtual pitches from those spectral pitches through a process of subharmonic matching [14]. Terhardt explained that "both spectral pitch and virtual pitch ultimately are dependent on aural Fourier analysis [but spectral] pitch is the most important carrier of auditory information, as it is an element of higher-order, Gestalt-like types of auditory precepts…" [15].

Terhardt and his colleagues expanded on the relevant auditory processing principles in relation to a specific chord:

> In the case of a major triad played on a piano, the tonal precept consists of (1) several spectral pitches which correspond to individual harmonics of the piano's complex tones, (2) several virtual pitches which correspond to the chord's musical notes, and (3) some additional virtual pitches in the frequency region below the lowest fundamental of the lowest chord note; these virtual pitches correspond to the major triad's "fundamental note" in a musical sense, i.e. the harmonic root [16].

Of course, in the case of a chord played on the piano, the tonal precept is shaped by the timbre of the piano as performed in a specific acoustic environment. We shall discuss timbre in more detail later. It should also be clear that the individual harmonics of the piano's complex tones includes the chord's musical notes.

1.2.3 Terhardt's simplified algorithm and initial revisions

With respect to the method set out in this book, Terhardt's work is important for his attempt to create a simplified algorithm that would reveal the roots of chords. He believed that the simplified algorithm could explain the psychophysical foundations of harmony.

Terhardt described his simplified algorithm as follows:

"Fortunately (for the theory of harmony), the pitch patterns elicited by harmonic complex tones are not too heavily dependent on the shape of the Fourier spectrum, i.e., timbre. So, if one presupposes - as a kind of general convention - that musical tones throughout are to be realized as harmonic complex tones, one can indeed represent them by note symbols, at least to a reasonable approximation.

Based on this convention, one can deduce from the theory of virtual pitch a root-finding algorithm that operates on the score - just as in conventional music theory […]. The principle is, that all candidates of roots must be subharmonics of the spectral pitches elicited by the actual sound, and that the prominence of any root is enhanced by "subharmonic coincidence" […]. When one translates this concept into musical notation, one gets a simple algorithm that shall be explained by the following example.

Chord notes:	c	f	a
same	C	**F**	A
- 1 fifth	**F**	B♭	D
- 1 mj 3rd	A♭	D♭	**F**
+ 1 mj 2nd	D	G	B
- 1 mj 2nd	B♭	E♭	G

The sample chord of which the root(s) are to be determined is written into the first line (2nd to 4th column, lowercase letters), i.e., c-f-a. The root candidates derived from the notes are written into the pertinent columns (uppercase letters) [.] The first candidate corresponds to both the first and second subharmonics, i.e., it is just the same note (second line), because, by definition, octave-equivalent pitches are not distinguished. The second candidate (third line), corresponding to the third subharmonic, is obtained by stepping one fifth down from the chord note, as octaves are not distinguished. The third candidate (fourth line), corresponding to the 5th subharmonic (the fourth is omitted, due to octave-equivalence) is obtained by stepping one major third down. The fourth candidate (fifth line), [corresponding] to the 7th subharmonic (the 6th is omitted, due to octave-equivalence to the third), is obtained by stepping either down by a minor seventh or stepping up one full tone (due to octave equivalence). The 8th subharmonic is omitted due to octave equivalence, such that the last candidate (6th line) corresponds to the 9th subharmonic, and it is obtained by either stepping up a 9th interval or stepping down one full tone (octave equivalence). As one can see in the table, there is one and only one candidate, namely F, that occurs in all three columns (full subharmonic match). This, by definition, indicates that F is the most prominent root of the chord.

As is apparent from this example, the determination of root candidates cannot fail for any type of chord. What may happen just is that there is no "full match", i.e., that there does not occur any candidate which is found in all columns, such that the root is less pronounced and more ambiguous than in a major triad such as above.

As another example let us consider a chord that by conventional theory has been regarded to be "at the borderline to atonality", i.e., the famous Tristan chord. Here is the corresponding table:

Tristan chord:	f	b	d♯	g♯
	F	B	D♯	G♯
	A♯	E	G♯	**C♯**
	C♯	G	B	E
	G	**C♯**	F	A♯
	D♯	A	**C♯**	F♯

The algorithm tells us that actually there is a full match for the root C♯. Thus the chord, when considered in isolation, is far from being atonal. One can easily verify that the root C♯ indeed "makes sense", i.e., by playing it in the bass register together with the Tristan chord. So, the subharmonic matching algorithm has found out what one may as well explain in terms of the conventional theory: The Tristan chord f-b-d♯-g♯ can be said to be a major 9th chord with root C♯, of which the root note itself is missing.

In summary, I believe that with the above concept of harmony, and with the explanations outlined for tone affinity and root-relationship, the psychophysical foundations of harmony are laid." [17].

Note that Terhardt stated that “the pitch patterns elicited by harmonic complex tones are not too heavily dependent on […] timbre”. This statement implies that harmonic complex tones are dependent on timbre to some degree. He also indicated that the prominence of any root is enhanced by "subharmonic coincidence". He also touched on the concept of root ambiguity. We will return to these concepts later, and show how they can be used to improve our understanding of the sound of chords.

The simplified algorithm, which Terhardt discussed above, can be set out in the form of a chart (Figure 1.5).

Spectral Pitches (Harmonics) and Virtual Root Candidates (Subharmonics)

	Intervals	Semitones												
Spectral Pitches (Harmonics)	M2 up	2 up	D	D♯ E♭	E	E♯ F	F♯ (G♭)	G	G♯ A♭	A	A♯ B♭	B	B♯ C	C♯ (D♭)	9th harmonic
	m7 up	10 up	B♭ (A♯)	B	C	C♯ D♭	D	E♭ (D♯)	E	F	F♯ G♭	G	G♯ A♭	A	7th harmonic
	M3 up	4 up	E	E♯ F	F♯ (G♭)	F♯♯ G	G♯ (A♭)	A	A♯ B♭	B	B♯ C	C♯ (D♭)	C♯♯ D	D♯ (E♭)	5th harmonic
	P5 up	7 up	G	G♯ A♭	A	A♯ B♭	B	C	C♯ D♭	D	D♯ E♭	E	E♯ F	F♯ (G♭)	3rd and 6th harmonics
	P8 up	12 up	C	C♯ D♭	D	D♯ E♭	E	F	F♯ G♭	G	G♯ A♭	A	A♯ B♭	B	2nd, 4th and 8th harmonics
Performed Chord Notes			C	C♯ D♭	D	D♯ E♭	E	F	F♯ G♭	G	G♯ A♭	A	A♯ B♭	B	
			0 - 12	**1 - 11**	**2 - 10**	**3 - 9**	**4 - 8**	**5 - 7**	**6 - 6**	**7 - 5**	**8 - 4**	**9 - 3**	**10 - 2**	**11 - 1**	1st harmonic 1st subharmonic
Virtual Root Candidates (Subharmonics)	P8 down	12 down	C	C♯ D♭	D	D♯ E♭	E	F	F♯ G♭	G	G♯ A♭	A	A♯ B♭	B	2nd, 4th and 8th subharmonics
	P5 down	7 down	F	F♯ G♭	G	G♯ A♭	A	B♭ (A♯)	B	C	C♯ D♭	D	D♯ E♭	E	3rd and 6th subharmonics
	M3 down	4 down	A♭ (G♯)	A	B♭ (A♯)	B	C	D♭ (C♯)	D	E♭ (D♯)	E	F	F♯ G♭	G	5th subharmonic
	m7 down	10 down	D	D♯ E♭	E	E♯ F	F♯ (G♭)	G	G♯ A♭	A	A♯ B♭	B	B♯ C	C♯ (D♭)	7th subharmonic
	M2 down	2 down	B♭ (A♯)	B	C	C♯ D♭	D	E♭ (D♯)	E	F	F♯ G♭	G	G♯ A♭	A	9th subharmonic

Figure 1.5 Terhardt's simplified algorithm.

Terhardt's simplified algorithm can be modified to make it function at a more abstract level by replacing the letter names and accidentals with numbers from 0 to 11 (Figure 1.6).

Spectral Pitches (Harmonics) and Virtual Root Candidates (Subharmonics)

	Intervals	Semitones													
Spectral Pitches (Harmonics)	M2 up	2 up	2	3	4	5	6	7	8	9	10	11	0	1	9th harmonic
	m7 up	10 up	10	11	0	1	2	3	4	5	6	7	8	9	7th harmonic
	M3 up	4 up	4	5	6	7	8	9	10	11	0	1	2	3	5th harmonic
	P5 up	7 up	7	8	9	10	11	0	1	2	3	4	5	6	3rd and 6th harmonics
	P8 up	12 up	0	1	2	3	4	5	6	7	8	9	10	11	2nd, 4th and 8th harmonics
Performed Chord Notes			**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	
			0 - 12	**1 - 11**	**2 - 10**	**3 - 9**	**4 - 8**	**5 - 7**	**6 - 6**	**7 - 5**	**8 - 4**	**9 - 3**	**10 - 2**	**11 - 1**	1st harmonic 1st subharmonic
Virtual Root Candidates (Subharmonics)	P8 down	12 down	0	1	2	3	4	5	6	7	8	9	10	11	2nd, 4th and 8th subharmonics
	P5 down	7 down	5	6	7	8	9	10	11	0	1	2	3	4	3rd and 6th subharmonics
	M3 down	4 down	8	9	10	11	0	1	2	3	4	5	6	7	5th subharmonic
	m7 down	10 down	2	3	4	5	6	7	8	9	10	11	0	1	7th subharmonic
	M2 down	2 down	10	11	0	1	2	3	4	5	6	7	8	9	9th subharmonic

Figure 1.6 Terhardt's simplified algorithm with letter names and accidentals replaced by numbers from 0 to 11.

In Figure 1.6, transpositional twelve-point circular space is represented as a single line of performed chord notes (0… 11), but the ends are actually connected to form a circular space (so, for example, 11 + 1 = 0). In acoustics (and psychoacoustics), the harmonic series (and spectral pitches) rise above the performed chordal pitches over more than one octave. In our method, octave equivalence applies and the spectral pitches are represented by numbers in transpositional twelve-point circular space. Likewise, octave equivalence is applied to subharmonics. Each harmonic (spectral pitch) and subharmonic level is also circular. Figure 1.7 shows the performed chord notes and their spectral pitches in transpositional twelve-point circular space.

In this method, the term "spectral pitch" is used as it clarifies the relationship between the psychoacoustic level and our mathematical formula. However, all calculations made with this formula in relation to spectral pitches produce results that are numbered points in transpositional twelve-point circular space.

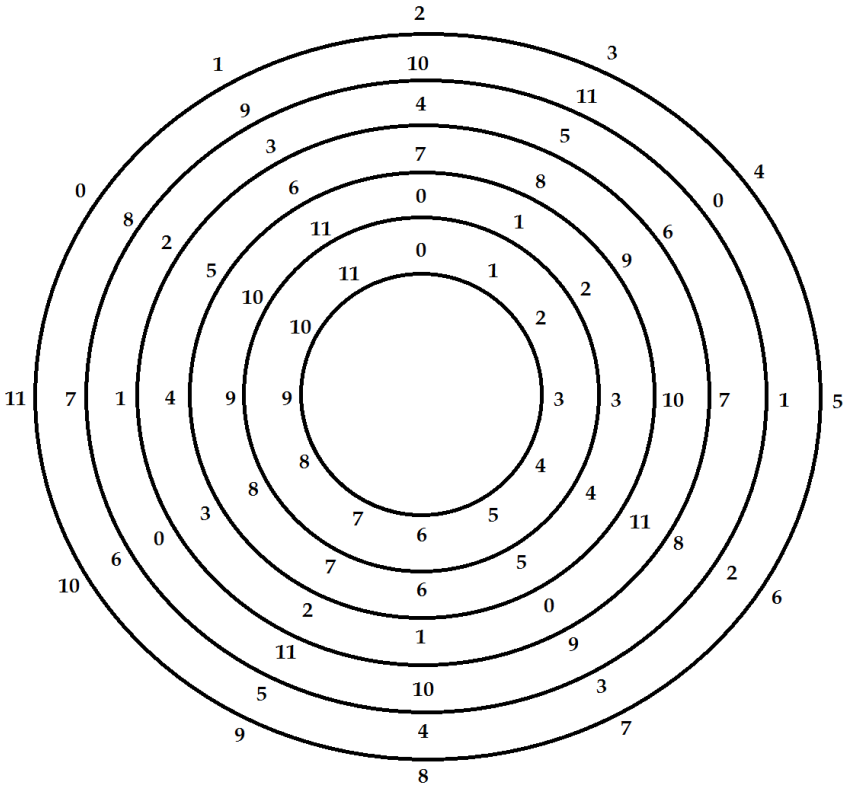


Figure 1.7 Performed chord notes circled by their spectral pitches in transpositional twelve-point circular space.

Terhardt assumed that the pitch patterns elicited by harmonic complex tones are not too heavily dependent on timbre, and concluded that it is reasonable to ignore harmonic amplitude. However, it appears that the amplitudes of the fundamental and all harmonics, and the strength of subharmonics, have more significant impact on the pitch patterns than he acknowledged.

Based on the chords that Terhardt has used as examples of the application of his simplified algorithm, a processed chord implicitly contains at least three performed chordal notes (although there does not seem to be any specific exclusion of intervals). Of course, Terhardt's simplified algorithm is not intended to identify virtual root candidates for a single pitch as there must be more than one chordal pitch to allow for the analysis of subharmonic pitch matches.

Nonetheless, for a single pitch, Terhardt's simplified algorithm can be used to identify five, equal unordered spectral pitches (0, 2, 4, 7 and 10) and five, equal unordered root candidates (0, 2, 5, 8 and 10) in transpositional twelve-point circular space. Identifying these pitches is an important first step in evaluating the sound of chords. However, these results are inconsistent with conventional music theory, which does not conclude that a single pitch has five equal roots. Treating all spectral pitches as equal, and all root

candidates as equal, is inconsistent with acoustics and psychoacoustics; this approach does not reflect how people hear. Generally speaking, harmonics that are more distant from a fundamental pitch along the harmonic series tend to reverberate more quietly than less distant harmonics.

Applying Terhardt's simplified algorithm to a perfect fifth interval reveals a problem underlying an unweighted pattern matching approach.

For the perfect fifth interval (0-7), Terhardt's simplified algorithm identifies two virtual roots, 0 and 5 (Figure 1.8). Each one is selected because 2 pitches match in the subharmonic series for the chordal pitches, 0 and 7. In Figure 1.8, the root candidates 0 and 5 are equal and unordered, and 2, 3, 7, 8, 9 and 10 are another subset of equal, unordered root candidates. In other words, no prominence is implied between the two virtual roots or amongst the other root candidates, except that the two virtual roots are more prominent than the rest.

Root candidates	0	5	2	3	7	8	9	10
Number of pitches that match	2	2	None	None	None	None	None	None
Terhardt's simplified virtual root	VR	VR	-	-	-	-	-	-

Figure 1.8 Terhardt's simplified algorithm in Figure 1.6 applied to the root candidates of a perfect fifth interval (0-7).

These results are inconsistent with conventional music theory, which does not conclude that a perfect fifth has two equal roots or that 5 is the root of 0-7. These results do not appear to reflect what people hear.

Terhardt's simplified pattern-matching algorithm can also provide anomalous results for some triads.

For the A minor triad in root position (A 0-3-7), first inversion (C 0-4-9), and second inversion (E 0-5-8), Terhardt's simplified algorithm identifies one VR in each case, (5 = D; 2 = D; and 10 = D). This is inconsistent with the analysis by Terhardt and others of the virtual roots of A minor triads played on a piano, which showed that the virtual root of the A minor triad in root position (A_4, C_5, E_5) was A_4 (= 0 in transpositional twelve-point circular space) and in first inversion (C_5, E_5, A_5) was A_4 (= 9) (among other contenders). It is also inconsistent with their results for the A minor triad in second inversion (E_5, A_5, C_6), which had two ambiguous equally-weighted virtual roots (E_4 and A_3 = 0 and 5) (again, among other contenders) [18]. As well, the results produced by

Terhardt's simplified algorithm for a minor triad are also inconsistent with conventional music theory and do not appear to reflect what people hear.

It appears that the Terhardt's simplified pattern-matching algorithm errs on the side of being too simplified, and provides misleading results in some cases. How can Terhardt's simplified pattern-matching algorithm be modified to be more accurate while minimizing the incorporation of acoustic or psychoacoustic variables?

1.2.4 Timbre weighting of spectral pitches and root candidates

As complex harmonic tones are dependent on timbre to some extent, a method that takes this fact into account may be able to produce more realistic and practical results than one that does not.

However, timbre is very complex. Every voice has a distinct timbre, which can change when one speaks, sings or whispers. Every type of instrument has different timbres from other instruments. Timbre changes as pitches are played in different octaves on the same instrument. Henry Brandt has concluded that there are thirteen different groups of closely-related "live" acoustic (rather than electronic, amplified or synthesized) instrumental timbres [19]. Likewise, timbre can change with how a single pitch is played, such as articulations and dynamics (including crescendos).

There is no perfect approach for determining the range of pitches to be used or the frequency of each harmonic to be used. However, the approach chosen should be reasonable, simple and workable, as the goal is to help young composers by providing them with a practical method for evaluating and comparing the sound of chords at the sensory level.

The human ear evolved along with the human voice, including singing, long before the development of sophisticated musical instruments. Furthermore, as Gill and Purves indicate, "[…] most music, even purely instrumental music, is composed within the human vocal range, and some popular instruments (e.g., the violin) bear a timbral resemblance to the human voice." [20]

Reflecting the range of the human voice to some degree would be appropriate. The range between the lowest notes of a basso cantante to the highest notes of a lyric soprano is approximately 87.31 Hz [F_2] to 1046.50 Hz [C_6].

However, this range is too wide for a single voice. Using the timbres of several voices (soprano, alto, tenor and bass) would introduce anomalies, as the frequency patterns of the harmonics for each singer's voice would differ, even when singing the same pitch. Likewise, selecting the timbres of different acoustic orchestral instruments would also introduce anomalies, whether in the same octave or different octaves.

The approach chosen for this method is to create a generic timbre that is a proxy for vocal and acoustic timbres generally and is not intended to represent the timbre of any specific note sung by any voice or played on any acoustic orchestral instrument under any circumstances. However, it is intended to be a reasonable timbre in the sense that it could represent the harmonics of many notes sung by a voice or played by an acoustic orchestral instrument (excluding percussions instruments) in the range between F_2 and C_6 [21].

To create the generic timbre used in this method, I have weighted each harmonic and subharmonic by arbitrarily dividing each value by a series of numbers from the Fibonacci series (Figure 1.9). It is important to be aware that *all* of the values derived from this method - other than those for harmonic similarity (hsim) - are directly influenced by the timbre weight percentages. Different percentages would produce different results [22]. This relationship is most easily observed in the weighting of the spectral pitches for a single pitch, which equal the percentages used in the generic timbre (Figure 1.10).

Harmonic / subharmonic	**Unweighted value**		**Fibonacci divisor**		**Percent**
2	1	/	1	=	100.00
3	1	/	2	=	50.00
5	1	/	3	=	33 1/3
7	1	/	5	=	20.00
9	1	/	8	=	12.50

Figure 1.9 Generic timbre weightings derived with Fibonacci series values.

Spectral pitches	0	5	8	2	10
Timbre-weighted spectral pitch values	100.00	50.00	33.33	20.00	12.50

Figure 1.10 Spectral pitch values for a single pitch (0) weighted by the generic timbre.

I have clarified Terhardt's pattern matching approach to focus on the harmonics and subharmonics as set out in Figure 1.11:

The 9th harmonic
The 7th harmonic
The 5th harmonic
The 3rd or 6th harmonic, whichever has the largest amplitude
The note played or the 2nd, 4th or 8th harmonic, whichever has the largest amplitude

The note played or the 2nd, 4th or 8th subharmonic, whichever has the largest amplitude
The 3rd or 6th subharmonic, whichever has the largest amplitude
The 5th subharmonic
The 7th subharmonic
The 9th subharmonic

Figure 1.11 Clarified harmonic and subharmonic frequencies.

So, the weightings from the generic timbre for the harmonics and subharmonics in this method are set out in Figure 1.12.

Harmonics and subharmonics	**Percent**
The 9th harmonic	12.50
The 7th harmonic	20.00
The 5th harmonic	33 1/3
The 3rd or 6th harmonic, whichever has the largest amplitude	50.00
The note played or the 2nd, 4th or 8th harmonic, whichever has the largest amplitude	100.00
The note played or the 2nd, 4th or 8th subharmonic, whichever has the largest amplitude	100.00
The 3rd or 6th subharmonic, whichever has the largest amplitude	50.00
The 5th subharmonic	33 1/3
The 7th subharmonic	20.00
The 9th subharmonic	12.50

Figure 1.12 The weightings from the generic timbre applied to harmonics and subharmonics.

When the preferences and percentages in Figure 1.12 are integrated into Terhardt's simplified algorithm as shown in Figure 1.6, the modified algorithm takes the form set out in Figure 1.13 below.

Spectral Pitches (Harmonics) and Root Candidates (Subharmonics)

	Percent													
Spectral Pitches (Harmonics)	12.50	2	3	4	5	6	7	8	9	10	11	0	1	9th harmonic
	20.00	10	11	0	1	2	3	4	5	6	7	8	9	7th harmonic
	33.33	4	5	6	7	8	9	10	11	0	1	2	3	5th harmonic
	50.00	7	8	9	10	11	0	1	2	3	4	5	6	The 3rd or 6th harmonic, whichever has the largest amplitude
	100.00	0	1	2	3	4	5	6	7	8	9	10	11	The note played or the 2nd, 4th or 8th harmonic, whichever has the largest amplitude
Performed Chord Notes		**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	
		0 - 12	**1 - 11**	**2 - 10**	**3 - 9**	**4 - 8**	**5 - 7**	**6 - 6**	**7 - 5**	**8 - 4**	**9 - 3**	**10 - 2**	**11 - 1**	1st harmonic 1st subharmonic
Root Candidates (Subharmonics)	100.00	0	1	2	3	4	5	6	7	8	9	10	11	The note played or the 2nd, 4th or 8th subharmonic, whichever has the largest amplitude
	50.00	5	6	7	8	9	10	11	0	1	2	3	4	The 3rd or 6th subharmonic, whichever has the largest amplitude
	33.33	8	9	10	11	0	1	2	3	4	5	6	7	5th subharmonic
	20.00	2	3	4	5	6	7	8	9	10	11	0	1	7th subharmonic
	12.50	10	11	0	1	2	3	4	5	6	7	8	9	9th subharmonic

Figure 1.13 The timbre-weighted algorithm, with weightings from the generic timbre.

For processing the spectral pitches of chords, including their weights, this timbre-weighted algorithm represents the final form of our method. It is used to calculate the spectral pitches and weights set out in Appendix A (The Method for Evaluating and Comparing the Sound of Chords).

For example, the spectral pitch series shown in the columns above the performed chordal notes 0 and 7 both contain the spectral pitch 7. In one instance, the spectral pitch weight is 100.00%; in the other, it is 50.00%. For the perfect fifth interval (0-7), the spectral pitch 7 has a spectral pitch weight of 150.00%.

However, the percentage weighting of the subharmonics changes Terhardt's modified algorithm from being a virtual pitch extraction model and turns it into a timbre-weighted root candidate model that does not identify virtual pitches. As we will show below, further modification will be required to take subharmonic coincidence into account.

Applying Terhardt's simplified algorithm to a minor triad in root position (0-3-7) identifies a false virtual root, 5 (Figure 1.14).

Root candidates	5	0	3	8	1	2	7	9	10	11
Number of pitches that match	3	2	2	2	None	None	None	None	None	None
Terhardt's simplified virtual root	VR	-	-	-	-	-	-	-	-	-

Figure 1.14 Terhardt's simplified algorithm in Figure 1.6 applied to a minor triad in root position (0-3-7).

This issue is resolved when the timbre-weighted algorithm in Figure 1.13 is applied (see Figure 1.15). The lowest pitch in the stack of thirds (0) becomes the root, consistent with conventional music theory. In addition, the prominence of the root candidates declines in a manner which reflects that harmonics, which are more distant from a fundamental pitch along the harmonic series, tend to reverberate more quietly than less distant harmonics. This spectral pattern is mirrored in the timbre-weighting of the root candidates. In this case, two equal unordered pairs of root candidates are listed, 2 and 9, and 1 and 10.

Root candidates	0	3	7	8	5	11	2	9	1	10
Timbre-weighted root-candidate values	150.00	133.33	100.00	83.33	82.50	33.33	20.00	20.00	12.50	12.50
Terhardt's simplified virtual root	-	-	-	-	VR	-	-	-	-	-

Figure 1.15 The timbre-weighted algorithm in Figure 1.13 applied to a minor triad in root position (0-3-7).

However, applying the timbre-weighted algorithm, by itself, can create problems of its own.

For example, applying Terhardt's simplified algorithm to a Tristan chord (0-3-6-10) (Figure 1.16) identifies the true virtual root, 8, as discussed by Terhardt above. For this chord, the prominence of the one virtual root is consistent with my subjective listening tests [23].

Root candidates	8	0	2	3	5	6	10	11	1	4
Number of pitches that match	4	2	2	2	2	2	2	2	None	None
Terhardt's simplified virtual root	VR	-	-	-	-	-	-	-	-	-

Figure 1.16 Terhardt's simplified algorithm in Figure 1.6 applied to a Tristan chord (0-3-6-10).

However, when the timbre-weighted algorithm in Figure 1.13 is applied, the prominence of the virtual root is lost (Figure 1.17).

Root candidates	3	6	0	8	10	11	5	2	1	4
Timbre-weighted root-candidate values	150.00	133.33	120.00	115.83	112.50	83.33	70.00	53.33	12.50	12.50
Terhardt's simplified virtual root	-	-	-	VR	-	-	-	-	-	-

Figure 1.17 The timbre-weighted algorithm in Figure 1.13 applied to a Tristan chord (0-3-6-10).

Therefore, our method needs to be revised to take into account how the brain processes subharmonic matches that are perceived as virtual roots.

1.2.5 The subharmonic enhancement multiplier

As Terhardt indicated above, the prominence of any root is enhanced by subharmonic coincidence. The approach that we use in this method to represent subharmonic coincidence is to apply a formula that increases the timbre-weighted root-candidate values of each set of pitches (*as a numbered point set*) that have subharmonic matches, with results that vary according to the number of matches for that pitch set and the subharmonic enhancement percentage applied.

Over the next two pages, the term "pitch" is used as it may clarify the relationship between subharmonic coincidences at the psychoacoustic level and our mathematical formula. However, all calculations made with this formula produce results that are numbered points in transpositional twelve-point circular space.

Let x = the subharmonic enhancement multiplier.

Let x = 1 + (y * z).

Let y = pitch multiplier for the number of matches for each pitch. If no pitches match, y = 0.

Let z = the subharmonic enhancement percentage.

For each performed pitch in a chord, polygon diagrams can be used to produce the pitch multipliers for the number of pitches among the subharmonic series that match. The first three polygons in the series are shown below (Figure 1.18). The dots represent the number of pitches that match, and the lines represent the number of matches, which are used as pitch multipliers (y).

Figure 1.18 Polygon diagrams.

With one exception, the values produced by the polygon diagrams are identical to the values which were used by Gill and Purves as divisors in their formula that established the harmonic similarity of scales to the harmonic series (Figure 1.19). The exception is that the series used in this method starts with the number 2 rather than the number 1 because there must be at least two identical pitches to have a match.

It is important to be aware that this polygon model differs from the formulas set out in Terhardt's complete psychoacoustic model, which relies on the application of physics to the frequencies of performed pitches measured in Hz [24].

How many pitches match?	2	3	4	5	6	7	8	9	10	11
Pitch multiplier (y)	1	3	6	10	15	21	28	36	45	55

Figure 1.19 Pitch multipliers (y).

If 2 pitches match each other, y = 1; if 3 pitches match each other, y = 3; if 4 pitches match each other, y = 6; and so on.

These pitch multipliers are used for every set of matches that result from performing a chord. For example, if a 4-pitch chord has one root candidate with 4 pitches that match each other, and a second root candidate with 2 pitches that match each other, the pitch multiplier 6 applies to the first root candidate and the pitch multiplier 1 applies to the second root candidate.

What should the subharmonic enhancement percentage be?

My unpublished analysis of various selected chords suggests that a subharmonic enhancement percentage of 20% is close to ideal overall, but perhaps a percentage somewhat less than 20% would be best. The analysis considered alternative percentages from 10% to 70%.

My subjective listening to the same selection of chords suggests that a 20% subharmonic enhancement percentage is better than 10%, so a percentage closer to 20% than to 10% is best overall. By "better", I mean that the use of the subharmonic enhancement percentage allows the method to identify root candidates in a descending order that best reflects their sense of being the root of the chord (from most likely to be the root to least likely) when played one, two and three octaves below. Subjectively, the most likely root could occur in any one of those octaves (or in more than one octave).

First, let's consider the Tristan chord again (see Figure 1.16 and Figure 1.17).

When the subharmonic enhancement multiplier is applied to the timbre-weighted root candidates using a subharmonic enhancement percentage of 10.0%, the root candidate with the highest weight is once again the virtual root 8 (Figure 1.20). However, the root candidate value for 8 is only somewhat greater than that for 3, which suggests that there is a low degree of clarity between these two root candidates, which is not consistent with subjective listening.

Root candidates	8	3	6	0	10	11	5	2	1	4
Root-candidate values with z = 10%	185.33	165.00	146.67	132.00	123.75	91.67	77.00	58.67	12.50	12.50
Terhardt's simplified virtual root	VR	-	-	-	-	-	-	-	-	-

Figure 1.20 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 10.0%, applied to a Tristan chord (0-3-6-10).

When the subharmonic enhancement percentage is increased to 20.0%, there is considerably more clarity between the first two root candidates (Figure 1.21).

Root candidates	8	3	6	0	10	11	5	2	1	4
Root-candidate values with z = 20%	254.83	180.00	160.00	144.00	135.00	100.00	84.00	64.00	12.50	12.50
Terhardt's simplified virtual root	VR	-	-	-	-	-	-	-	-	-

Figure 1.21 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 20.0%, applied to a Tristan chord (0-3-6-10).

So, in order for the Tristan chord to have its virtual root 8 as the highest weighted and selected root, the subharmonic enhancement percentage must be at least 10%. However, as subjective listening suggests that 8 is clearly the root of this chord, a higher subharmonic enhancement percentage is required to reflect a greater degree of root clarity relative to the other root candidates. So, subjective listening suggests that a percentage closer to 20% than to 10% would be appropriate.

Second, let's consider the minor triad in root position once more (see Figure 1.14 and Figure 1.15).

When the subharmonic enhancement multiplier is applied to the timbre-weighted root candidates using a subharmonic enhancement percentage of 10.0%, the root candidate with the highest weight remains 0, but the virtual root 5 is now weighted somewhat more highly than root candidate 7 as well as more highly than 8 but less than 0 and 3 (Figure 1.22).

Root candidates	0	3	5	7	8	11	2	9	1	10
Root-candidate values with z = 10%	165.00	146.67	107.25	100.00	91.67	33.33	20.00	20.00	12.50	12.50
Terhardt's simplified virtual root	-	-	VR	-	-	-	-	-	-	-

Figure 1.22 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 10.0%, applied to a minor triad in root position (0-3-7).

When the subharmonic enhancement percentage is increased to 20.0%, the virtual root 5 is now weighted significantly higher than root candidate 7 (Figure 1.23). Subjective listening suggests that this weight is too high and that a subharmonic enhancement percentage of 20.0% is too large.

Root candidates	0	3	5	7	8	11	2	9	1	10
Root-candidate values with z = 20%	180.00	160.00	132.00	100.00	100.00	33.33	20.00	20.00	12.50	12.50
Terhardt's simplified virtual root	-	-	VR	-	-	-	-	-	-	-

Figure 1.23 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 20.0%, applied to a minor triad in root position (0-3-7).

When the subharmonic enhancement percentage is increased to 44.6%, the virtual root 5 becomes the second most highly-weighted root. Finally, when the subharmonic enhancement percentage is increased to 69.3%, the virtual root 5 becomes the most highly-weighted root. Clearly, these results are not consistent with conventional music theory and with widely-held perceptions of the root of the minor triad in root position.

So, in order to keep the 0-3-7 minor triad from having 5 as the highest-weighted or second-highest and selected root, the subharmonic enhancement percentage must be less than 44.6%.

For the 0-3-7 minor triad, subjective listening suggests that 0 is a clearly stronger root candidate than 5, which in turn is stronger than 8. This perception is consistent with the analysis by Terhardt and others of the virtual roots of A minor triads played on a piano [25]. Subjective listening also suggests that 0 is a clearly stronger root candidate than 3, which in turn is stronger than all other root candidates; but the predominance of 7 over 5 and vice-versa depends on the octave in which those root candidates are played in. These perceptions are not consistent with the analysis by Terhardt and others, which did not report 3, 7 or 5 as being virtual root candidates. Overall, subjective listening suggests that a subharmonic enhancement percentage that is closer to 10% than to 20% would be appropriate.

A perfect subharmonic enhancement percentage does not appear to exist. The best one can do is to select a percentage that provides reasonable root-candidate values for a broad range of chords. Upward or downward shifts in the percentage may provide more reasonable values for the root candidates of some chords but less reasonable values for others. The impact of the percentage on the value of the first root candidate for a chord is relatively more important than its impact on the value of the second root candidate, which is relatively more important than its impact on the third root candidate, and so on. The optimal percentage to be used with all chords is ultimately somewhat of a compromise. It is an exercise of judgement.

Arbitrarily, I have chosen 16.66666666666667% (= 1/6) to be the subharmonic enhancement percentage (z) for the purposes of this method.

Consequently, the following subharmonic enhancement multipliers in Figure 1.24 apply. In practice, there are unlikely to be any matches that consist of large numbers of pitches.

How many pitches match?	2	3	4	5	6	7	8	9	10	11
Subharmonic enhancement multiplier (x)	1.167	1.5	2.0	2.667	3.5	4.5	5.667	7.0	8.5	10.167

Figure 1.24 Subharmonic enhancement multipliers (x).

When the subharmonic enhancement percentage is set to 16.66666666666667% and the subharmonic enhancement multipliers are applied to a minor triad in root position (0-3-7), the virtual root 5 is now weighted somewhat more closely to root candidate 7 than it is when the subharmonic enhancement percentage is set to 20% (Figure 1.25).

Root candidates	0	3	5	7	8	11	2	9	1	10
Root-candidate values with z = 1/6	175.00	155.56	123.75	100.00	97.22	33.33	20.00	20.00	12.50	12.50
Terhardt's simplified virtual root	-	-	VR	-	-	-	-	-	-	-

Figure 1.25 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 16.66666666666667%, applied to a minor triad in root position (0-3-7).

The minor triad and the Tristan chord can be used again to illustrate the thought process behind the selection of 16.66666666666667% (= 1/6) to be the subharmonic enhancement percentage (z) for the purposes of this method. A somewhat lower percentage might be more appropriate to reflect the predominance of 7 and 5 as root candidates of a minor triad in root position. However, given the other constraints we have discussed, these root candidates are only vying for third and fourth place. On the other hand, this percentage clearly establishes the most-highly weighted root candidate for a Tristan chord (Figure 1.26).

Root candidates	8	3	6	0	10	11	5	2	1	4
Root-candidate values with z = 1/6	231.67	175.00	155.56	140.00	131.25	97.22	81.67	62.22	12.50	12.50
Terhardt's simplified virtual root	VR	-	-	-	-	-	-	-	-	-

Figure 1.26 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 16.66666666666667% (1/6), applied to a Tristan chord (0-3-6-10).

The timbre-weighted algorithm in Figure 1.13, with the root candidates enhanced by the subharmonic enhancement percentage set to 16.66666666666667%, provides the basis for calculating the root candidates of each chord, as set out in the third layer of Appendix A (The Method for Evaluating and Comparing the Sound of Chords).

The key point to understand is that the combined effect of the auditory processing of timbre and the production of subharmonic enhancement in the brain – both working

together – results in the perception of the roots of chords. Either of these processes alone is incomplete and may lead to inaccurate conclusions.

1.2.6 Root clarity and root ambiguity

As discussed above, for the perfect fifth interval (0-7), Terhardt's simplified algorithm identified two virtual roots, 0 and 5 (Figure 1.8). As previously noted, these results are inconsistent with conventional music theory, which does not conclude that a perfect fifth has two equal roots or that 5 is the root of 0-7; and these results do not appear to reflect what people hear. When the timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 16.66666666666667% is applied, the error produced by the simplified algorithm is resolved (Figure 1.27). The interval has one clear root, 0, which is consistent with conventional music theory and with what people hear. The root candidate values reveal that the 5 is a false virtual root.

Root candidates	0	7	5	3	8	2	9	10
Root-candidate values with z = 1/6	175.00	100.00	72.92	33.33	33.33	20.00	20.00	12.50
Terhardt's simplified virtual root	VR	-	VR	-	-	-	-	-

Figure 1.27 The timbre-weighted algorithm in Figure 1.13 and the subharmonic enhancement multiplier with a subharmonic enhancement percentage of 16.66666666666667% (1/6), applied to a perfect fifth interval (0-7).

The clarity of the root of a chord depends primarily on the difference between the root candidate values of the first and second root candidates. In this method, root clarity is a measure of the relative difference between the root-candidate values of the first root candidate (the highest root-candidate value) and the second root candidate, where the root-candidate value of the first root candidate is greater than that of the second. The greater the relative difference between the root-candidate values of the first and second root candidates, the greater the clarity of the root. If the root-candidate values of the first and second root candidates are identical, there is complete root ambiguity and no root clarity. This ambiguity increases if more than the first two root candidates have identical root candidate values. (Perceptually, this ambiguity would also increase if the root candidate value of the third root candidate is very close to the first two values.)

We use a simple formula to measure root clarity and root ambiguity.

Let a = the root clarity value.

Let a = (b - c) / c.

Let b = the root-candidate value of the first root candidate (the highest root-candidate value).

Let c = the root-candidate value of the second root candidate.

Figure 1.28 shows the root clarity value of a perfect fifth.

Root candidates	0	7
Root-candidate values with z = 1/6	175.00	100.00
Root clarity	75.00%	

Figure 1.28 The root clarity of a perfect fifth interval (0-7).

The root clarity of a single pitch can serve as a benchmark for comparing the root clarity values of other chords (Figure 1.29).

Root candidates	0	5
Root-candidate values with z = 1/6	100.00	50.00
Root clarity	100.00%	

Figure 1.29 The root clarity of a single pitch (0).

Also consider the root candidate values of major and minor triads in root position (Figure 1.30 and Figure 1.31).

Root candidates	0	4
Root-candidate values with z = 1/6	275.00	100.00
Root clarity	175.00%	

Figure 1.30 The root clarity of major triads in root position (0-4-7).

Root candidates	0	3
Root-candidate values with z = 1/6	175.00	155.56
Root clarity	12.50%	

Figure 1.31 The root clarity of minor triads in root position (0-3-7).

Conventional music theory and widely-held perceptions lead to the conclusion that major triads have significantly clearer and more stable roots than minor triads, and that the roots of minor triads are more ambiguous. The root clarity values produced by this method for the major and minor triads in root position are consistent with those conclusions.

The root clarity value is calculated using the root-candidate value of the first root candidate even if it is not a chordal pitch. This is illustrated in Figure 1.32 with the Tristan chord.

Root candidates	8	3
Root-candidate values with z = 1/6	231.67	175.00
Root clarity	32.38%	

Figure 1.32 The root clarity of the Tristan chord (0-3-6-10).

2
The Nature of Chords

2.1 What is the sound of a chord?

The sound of a chord consists, collectively, of several components:

- The performed pitches of the chord, which are represented in our method as a member of a transpositional set class;
- The harmonic similarity (hsim) of the performed pitches;
- The spectral pitches of the performed pitches, weighted by timbre;
- The root candidates of the chord, weighted by timbre and adjusted for subharmonic enhancement;
- The degree of root clarity or root ambiguity of the chord.

These components are set out in the first four layers of our method in Appendix A (The Method for Evaluating and Comparing the Sound of Chords).

The members of each transpositional set class (inversions) share the same unique set of performed pitches in transpositional twelve-point circular space.

The harmonic similarity of each chord tends to be unique when compared to other chords in the same transpositional set class or to other chords in different transpositional set classes. Nonetheless, a chord may have the same harmonic similarity as one or more other chords.

The members of each transpositional set class (inversions) share the same unique set of spectral pitches in transpositional twelve-point circular space.

A chord may have the same degree of root clarity or root ambiguity as other chords. Chords that are members of one transpositional set class, which is identified by the letter A or B, may share the same degree of root clarity or root ambiguity with chords that are members of another transpositional set classes that is related to it by inversion through reflection, and which is identified by the other letter (for example, see TSC3-4A and TSC3-4B in Appendix A (The Method for Evaluating and Comparing the Sound of Chords)). However, the degree of root clarity or root ambiguity between some transpositional set classes that are related by inversion through reflection may differ (for example, see: TSC3-8A and TSC3-8B; and TSC3-11A and TSC3-11B).

2.2 The harmonic similarity and root clarity of selected chords

It can be useful to compare the harmonic similarity and root clarity (or root ambiguity) of chords by plotting them a grid. The condensed charts below group chords together whose harmonic similarity and root clarity (or root ambiguity) fall within the ranges indicated. Where two or more chords fall within one cell, the harmonic similarity and root clarity of the chords in a cell are at least somewhat similar, but should not be presumed to be identical in any way.

Figure 2.1 is a condensed chart of the harmonic similarity and root clarity of single pitches (0) and of all intervals (0-1… 0-11).

Similarity to the harmonic series												
Most	to 100%							0				
	to 97.5%											
	to 95%											
	to 92.5%											
	to 90%											
	to 87.5%											
	to 85%											
	to 82.5%											
	to 80%											
	to 77.5%											
	to 75%											
	to 72.5%											
	to 70%											
	to 67.5%						0-7					
	to 65%											
	to 62.5%											
	to 60%											
	to 57.5%											
	to 55%											
	to 52.5%											
	to 50%						0-5					
	to 47.5%		0-9									
	to 45%											
	to 42.5%											
	to 40%					0-4						
	to 37.5%											
	to 35%		0-3									
	to 32.5%		0-6									
	to 30%			0-10		0-8						
	to 27.5%											
	to 25%											
	to 22.5%			0-2								
	to 20%		0-11									
	to 17.5%											
Less	to 15%											
	to 12.5%		0-1									
		0% three equal roots	0% two equal roots	>0% to 20%	>20% to 40%	>40% to 60%	>60% to 80%	>80% to 100%	>100% to 120%	>120% to 140%	>140% to 160%	>160% to 180%
		Ambiguous roots		Least ← **Root clarity** → Most								

Figure 2.1 Condensed chart of the harmonic similarity and root clarity of single pitches and intervals.

Figure 2.2 is a condensed chart of the harmonic similarity and root clarity of all major triads in root position and their inversions (0-4-7, 0-5-9 and 0-3-8), minor triads in root position and their inversions (0-3-7, 0-4-9 and 0-5-8), augmented triads (0-4-8),

diminished triads and their inversions (0-3-6, 0-3-9 and 0-6-9), the suspended triad (0-5-7) and its inversions (the Ilerici chord (0-2-7) and the perfect quartal trichord (0-5-10)), and the chromatic cluster (0-1-2).

Similarity to the harmonic series												
More	to 67.5%											
	to 65%											
	to 62.5%											
	to 60%											
	to 57.5%											
	to 55%											
	to 52.5%											
	to 50%											
	to 47.5%			0-3-7 0-4-9	0-2-7 0-5-7							0-4-7 0-5-9
	to 45%				0-5-10							
	to 42.5%											
	to 40%			0-5-8								0-3-8
	to 37.5%	0-4-8				0-3-9 0-6-9						
	to 35%					0-3-6						
	to 32.5%											
	to 30%											
	to 27.5%											
	to 25%											
	to 22.5%											
	to 20%											
	to 17.5%			0-1-2								
Less	to 15%											
	to 12.5%											
		0% three equal roots	0% two equal roots	>0% to 20%	>20% to 40%	>40% to 60%	>60% to 80%	>80% to 100%	>100% to 120%	>120% to 140%	>140% to 160%	>160% to 180%
		Ambiguous roots		Least ←				**Root clarity**				→ Most

Figure 2.2 Condensed chart of the harmonic similarity and root clarity of major, minor, augmented, diminished, and suspended triads (including the perfect quartal trichord and the Ilerici chord), and the chromatic cluster.

Figure 2.3 is a condensed chart of the harmonic similarity and root clarity of several chords that divide the octave evenly or nearly evenly.

The octave is divided evenly by only five types of chords: tritones (0-6), augmented triads (0-4-8), diminished seventh chords (0-3-6-9), whole-tone scales as chords (0-2-4-6-8-10), and the chromatic scale as a chord (0-1-2-3-4-5-6-7-8-9-10-11). All of these chords have ambiguous roots, and that ambiguity is more pronounced than their degree of

similarity to the harmonic series. The chromatic scale as a chord (0-1-2-3-4-5-6-7-8-9-10-11) has twelve equal roots and has the most ambiguous roots.

The octave is divided nearly evenly by perfect fourth intervals (0-5), perfect fifth intervals (0-7), minor triads in root position (0-3-7), minor triads in first inversion (0-4-9), minor triads in second inversion (0-5-8), major triads in root position (0-4-7), major triads in first inversion (0-3-8), major triads in second inversion (0-5-9), Tristan chords (0-3-6-10), major triad add six chords (0-4-7-9), dominant seventh chords (0-4-7-10), major seventh chords (0-4-7-11), pentatonic major scales as six ninth chords (0-2-4-7-9), dominant ninth chords (0-2-4-7-10), "Mystic" chords (0-2-4-6-9-10), dominant "sharp eleven" chords (0-2-4-6-7-10), and Dorian mode diatonic scales as chords (0-2-3-5-7-9-10). Other chords that divide the octave nearly evenly – such as rotations of chords with six or seven different pitches – are not shown.

Figure 2.3 shows that chords, which divide the octave nearly evenly, have greater similarity to the harmonic series and greater root clarity than chords that divide the octave evenly. This relationship holds true generally, but also applies specifically for all cardinalities (for all intervals, triads…).

Figure 2.3 also shows that there are significant differences among the harmonic similarity and root clarity of chords that divide the octave nearly evenly.

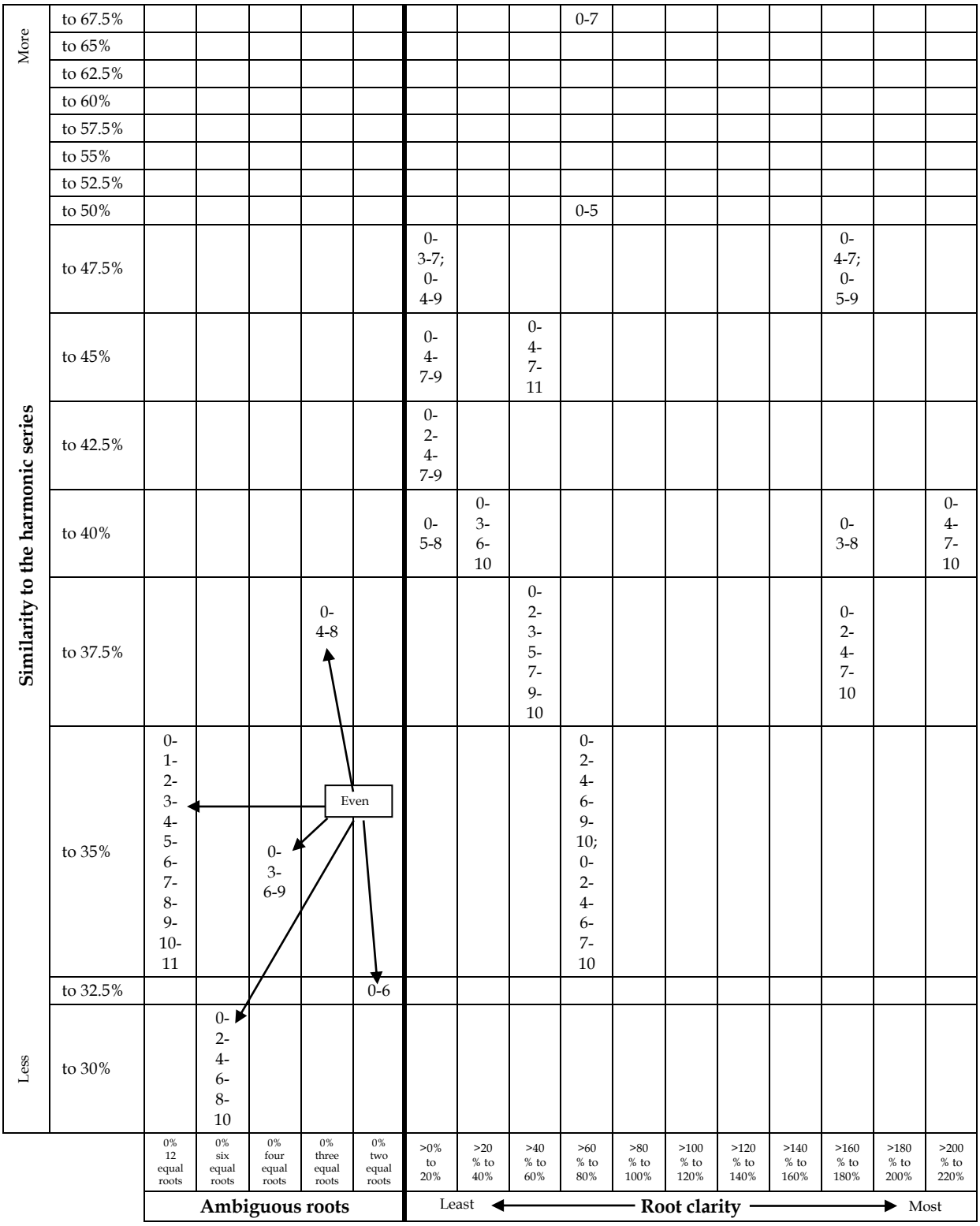


Figure 2.3 Condensed chart of the harmonic similarity and root clarity of selected chords that divide the octave evenly or nearly evenly.

2.3 What is the root position of a chord?

In every transpositional set class that does not have two or more ambiguous roots, there is one inversion in root position. It can be located in the corresponding "Root candidates and weights" chart for that transpositional set class in Appendix A. The first root candidate of the root position chord is always 0.

There are three exceptions to this basic, universal rule.

The first exception pertains to the five transpositional set classes that divide the octave evenly (TSC2-6 (0-6), TSC3-12 (0-4-8), TSC4-28 (0-3-6-9), TSC6-35 (0-2-4-6-8-10), and TSC12-1 (0-1-2-3-4-5-6-7-8-9-10-11)). The "Root candidates and weights" chart in Appendix A for these transpositional set classes must be interpreted carefully. The chart shows only one series of root candidates for that transpositional set class, and the first root candidate is always 0. In these five instances, the single series of root candidates is not intended to imply that there is only one root candidate for that transpositional set class. On the contrary, as each inversion of these chords rotates onto itself, every numbered point in the chord is competing equally to be in root position. In these five instances, the number of ambiguous inversions in root position is equal to the number of numbered points in the chord.

The second exception pertains to TSC3-10 (0-3-6, 0-3-9, and 0-6-9) and TSC4-27A (0-2-5-8, 0-3-6-10, 0-3-7-9, and 0-4-6-9). In these instances, the first root candidate of each chord is not a chordal pitch and is not 0. None of these chords are in root position. There may be other chords that fall within this exception.

The third exception applies to all other transpositional set classes that have two or more equal roots. The "Root candidates and weights" chart in Appendix A for these transpositional set classes will identify each inversion that is competing to be in root position. The first root candidate of each of these inversions is always 0. For an example, see TSC6-32 in Appendix A.

3
How to Use the Method for Evaluating and Comparing the Sound of Chords

3.1 The five layers

The sound of each chord illustrated in this book is expressed in Appendix A (The Method for Evaluating and Comparing the Sound of Chords)) in two graph layers and two chart layers. There is one additional name layer. Each of these layers identifies the chord by the performed chordal pitches set out in transpositional twelve-point circular space, and is grouped with the other members of its transpositional set class.

The first graph layer shows the similarity of each chord to the harmonic series. The degree of similarity is expressed as a percentage.

The first chart layer shows the spectral pitches and weights for each chord. The spectral pitches are shown in declining order as reflected in their diminishing weights. If two spectral pitches have the same weight, they should be treated as equivalent even though they are displayed in sequence.

The second graph layer shows the degree of root clarity or ambiguity of each chord. If the chord has a degree of root clarity, it is expressed as a percentage above the horizontal black line which represents 0%. Where the chord lacks root clarity and is ambiguous, the percentage is shown as 0% and the degree of root ambiguity is shown as falling into one of three categories: 2, 3 or 4 or more (4+) equal roots.

The second chart layer shows the root candidates and weights for each chord. The root candidates are shown in declining order as reflected in their diminishing weights. If two root candidates have the same weight, they should be treated as equivalent even though they are displayed in sequence.

The final layer in this method contains various names for chords. In some cases, the pitches in a chord may also be used as a scale. If so, the name of the scale may also be given. A geographic identifier may also be provided for specific names.

In every transpositional set class that does not have two or more ambiguous roots, the use of "first inversion" in a name is restricted to the first counter-clockwise rotation of the root position chord.

3.2 Evaluating and comparing the sound of chords

The degree of similarity of a chord to the harmonic series (hsim) appears to be closely related to the concepts of consonance and dissonance. Bowling and Purves have concluded that:

> In light of present evidence, the most plausible explanation for consonance and related tonal phenomenology is an evolved attraction to the harmonic series that characterize conspecific vocalizations, based on the biological importance of social sound signals. If correct, this explanation of consonance would rationalize at least some aspects of musical aesthetics. [26]

A high degree of harmonic similarity appears to be comparable to a high degree of consonance. Likewise, a low degree of harmonic similarity seems to be equivalent to a high degree of dissonance. The algorithm that determines the degree of similarity of a chord to the harmonic series provides a means of evaluating the sound of every possible chord in a manner that is free of cultural bias and the limitations of incomplete experimental data derived from human listeners.

The harmonic similarity of chords can be viewed and compared across all first graph layers in Appendix A. Collectively, if strung end to end, these layers look like the skyline of a large city.

For convenience, Figure 3.1 shows the harmonic similarity of all single pitches, two-pitch chords (intervals) and three-pitch chords (triads and trichords) in declining order. None of these chords has greater similarity to the harmonic series than single pitches.

Harmonic similarity percentages	Chords	Member of transpositional set class
100.00 %	**Single pitches (0)**	**TSC1-1**
66.67 %	Perfect fifths (0-7)	TSC2-5
50.00 %	Perfect fourths (0-5)	TSC2-5
46.67 %	Major sixths (0-9)	TSC2-3
	Minor triads in root position (0-3-7)	TSC3-11A
	Major triads in root position (0-4-7)	TSC3-11B
46.30 %	Add 9 omit 3 trichords (0-2-7)	TSC3-9
	Suspended triads (0-5-7)	
45.74 %	Third inversion dominant seventh sus 4s (0-2-7-9)	TSC4-23
45.56 %	First inversion minor triads (0-4-9)	TSC3-11A
	Second inversion major triads (0-5-9)	TSC3-11B
45.19 %	0-7-9 trichords (0-7-9)	TSC3-7A
	0-2-9 trichords (0-2-9)	TSC3-7B
44.82 %	Minor seventh tetrachords (0-3-7-10)	TSC4-26
44.17 %	Major seventh tetrachords (0-4-7-11)	TSC4-20
43.15 %	Second inversion major triad add sixes (0-2-5-9)	TSC4-26
	Major triad add sixes (0-4-7-9)	
42.96 %	Ute Tritonic scale as trichords (0-3-10)	TSC3-7A
	0-7-10 trichords (0-7-10)	TSC3-7B
	Perfect quartal trichords (0-5-10)	TSC3-9
41.85 %	Perfect quartal tetrachords (0-3-5-10)	TSC4-23
	Dominant seventh suspended fourths (0-5-7-10)	
41.67 %	0-4-11 trichords (0-4-11)	TSC3-4A
	0-7-11 trichords (0-7-11)	TSC3-4B
41.30 %	Third inversion major add 2 tetrachords (0-5-7-9)	TSC4-22A
	Third inversion Mistuned Historical Church Bell 8s (aural) (0-2-4-9)	TSC4-22B
40.74 %	Genus Primum scale as chords (0-2-5-7)	TSC4-23
40.65 %	Second inversion minor add 2 tetrachords (0-4-9-11)	TSC4-14A
	Second inversion major add 4 tetrachords (0-2-7-11)	TSC4-14B
40.05 %	Minor sixth tetrachords (0-3-7-9)	TSC4-27A
	Third inversion dominant seventh chords (0-2-6-9)	TSC4-27B
40.00 %	Major thirds (0-4)	TSC2-4
	Second inversion major seventh tetrachords (0-4-5-9)	TSC4-20
39.07 %	Major add 2 tetrachords (0-2-4-7)	TSC4-22A
	Minor add 4 tetrachords (0-3-5-7)	TSC4-22B
38.94 %	Half-diminished sevenths (0-3-6-10)	TSC4-27A

	Dominant sevenths (0-4-7-10)	TSC4-27B
38.89 %	Third inversion major seventh tetrachords (0-1-5-8)	TSC4-20
	First inversion major seventh tetrachords (0-3-7-8)	
38.52 %	Second inversion major add 2 tetrachords (0-3-8-10)	TSC4-22A
	Mistuned Historical Church Bell 8s (aural) (0-2-7-10)	TSC4-22B
38.15 %	First inversion minor add 2 tetrachords (0-1-5-10)	TSC4-14A
	Third inversion major add 4 tetrachords (0-5-9-10)	TSC4-14B
38.11 %	Major seventh plus four tetrachords (0-4-6-11)	TSC4-16A
	0-5-7-11 tetrachords (0-5-7-11)	TSC4-16B
38.06 %	Minor-major sevenths (0-3-7-11)	TSC4-19A
	Augmented major sevenths (0-4-8-11)	TSC4-19B
37.78 %	Second inversion minor triads (0-5-8)	TSC3-11A
	First inversion major triads (0-3-8)	TSC3-11B
	Major-minor tetrachords (0-3-4-7)	TSC4-17
37.59 %	Minor add 2 tetrachords (0-2-3-7)	TSC4-14A
	Major add 4 tetrachords (0-4-5-7)	TSC4-14B
37.28 %	0-1-5-7 tetrachords (0-1-5-7)	TSC4-16A
	0-2-6-7 tetrachords (0-2-6-7)	TSC4-16B
	0-4-6-9 tetrachords (0-4-6-9)	TSC4-27A
	Second inversion dominant seventh chords (0-3-5-9)	TSC4-27B
37.22 %	Third inversion minor-major sevenths (0-1-4-8)	TSC4-19A
	0-4-7-8 tetrachords (0-4-7-8)	TSC4-19B
37.14 %	0-6-7 trichords (0-6-7)	TSC3-5A
	0-1-7 trichords (0-1-7)	TSC3-5B
	First inversion diminished triads (0-3-9)	TSC3-10
	Second inversion diminished triads (0-6-9)	
37.13 %	0-2-9-11 tetrachords (0-2-9-11)	TSC4-10
36.91 %	0-1-7-8 tetrachords (0-1-7-8)	TSC4-8
36.72 %	0-3-9-10 tetrachords (0-3-9-10)	TSC4-13A
	Mistuned Historical Church Bell 7s (0-1-7-10)	TSC4-13B
36.67 %	0-7-8 trichords (0-7-8)	TSC3-4A
	0-1-8 trichords (0-1-8)	TSC3-4B
	Augmented triads (0-4-8)	TSC3-12
	First inversion minor-major sevenths (0-4-8-9)	TSC4-19A
	0-1-5-9 tetrachords (0-1-5-9)	TSC4-19B
36.63 %	0-4-5-11 tetrachords (0-4-5-11)	TSC4-8

	0-6-7-11 tetrachords (0-6-7-11)	
36.48 %	First inversion major triad add sixes (0-3-5-8)	TSC4-26
36.35 %	Diminished M7 chords (0-1-4-7)	TSC4-18A
	0-3-6-7 tetrachords (0-3-6-7)	TSC4-18B
36.02 %	0-2-4-11 tetrachords (0-2-4-11)	TSC4-11A
	0-7-9-11 tetrachords (0-7-9-11)	TSC4-11B
35.93 %	Third inversion minor add 2 tetrachords (0-5-7-8)	TSC4-14A
	First inversion major add 4 tetrachords (0-1-3-8)	TSC4-14B
35.79 %	Third inversion diminished M7 chords (0-5-6-9)	TSC4-18A
	0-3-4-9 tetrachords (0-3-4-9)	TSC4-18B
35.74 %	First inversion major add 2 tetrachords (0-2-5-10)	TSC4-22A
	Mistuned Historical Church Bell 8s (calculated) (0-5-8-10)	TSC4-22B
35.61 %	0-6-7-9 tetrachords (0-6-7-9)	TSC4-13A
	0-2-3-9 tetrachords (0-2-3-9)	TSC4-13B
	0-6-7-10 tetrachords (0-6-7-10)	TSC4-Z15A
	0-3-4-10 tetrachords (0-3-4-10)	TSC4-Z15B
	0-2-7-8 tetrachords (0-2-7-8)	TSC4-16A
	0-5-6-10 tetrachords (0-5-6-10)	
	0-4-5-10 tetrachords (0-4-5-10)	TSC4-16B
	0-1-6-8 tetrachords (0-1-6-8)	
35.56 %	First inversion major-minor tetrachords (0-1-4-9)	TSC4-17
	Third inversion major-minor tetrachords (0-5-8-9)	
35.19 %	0-1-3-10 tetrachords (0-1-3-10)	TSC4-10
	0-7-9-10 tetrachords (0-7-9-10)	
35.18 %	0-2-5 trichords (0-2-5)	TSC3-7A
	0-3-5 trichords (0-3-5)	TSC3-7B
35.05 %	0-2-8-9 tetrachords (0-2-8-9)	TSC4-Z15A
	0-1-7-9 tetrachords (0-1-7-9)	TSC4-Z15B
	0-4-9-10 tetrachords (0-4-9-10)	TSC4-Z29A
	0-1-6-10 tetrachords (0-1-6-10)	TSC4-Z29B
34.92 %	Diminished sevenths (0-3-6-9)	TSC4-28
34.78 %	Raga Sumukam scale as chords (0-2-6-11)	TSC4-Z29A
	0-5-9-11 tetrachords (0-5-9-11)	TSC4-Z29B
34.63 %	0-2-9-10 tetrachords (0-2-9-10)	TSC4-11A

	0-1-8-10 tetrachords (0-1-8-10)	TSC4-11B
34.50 %	All-interval tetrachord Bs (0-1-3-7)	TSC4-Z29A
	0-4-6-7 tetrachords (0-4-6-7)	TSC4-Z29B
34.44 %	0-1-5 trichords (0-1-5)	TSC3-4A
	0-4-5 trichords (0-4-5)	TSC3-4B
	Italian sixths (0-4-10)	TSC3-8A
	Second inversion major flat 5 trichords (0-6-10)	TSC3-8B
	Second inversion minor-major sevenths (0-4-5-8)	TSC4-19A
	0-3-4-8 tetrachords (0-3-4-8)	TSC4-19B
34.37 %	Messiaen's Truncated Mode 5 scale as chords (0-1-6-7)	TSC4-9
34.13 %	Second inversion diminished M7 chords (0-3-8-9)	TSC4-18A
	0-1-6-9 tetrachords (0-1-6-9)	TSC4-18B
33.66 %	0-2-5-11 tetrachords (0-2-5-11)	TSC4-13A
	0-6-9-11 tetrachords (0-6-9-11)	TSC4-13B
33.61 %	0-3-4-11 tetrachords (0-3-4-11)	TSC4-7
	0-7-8-11 tetrachords (0-7-8-11)	
33.39 %	0-2-5-8 tetrachords (0-2-5-8)	TSC4-27A
	First inversion dominant seventh chords (0-3-6-8)	TSC4-27B
33.33 %	Minor thirds (0-3)	TSC2-3
	0-1-8-9 tetrachords (0-1-8-9)	TSC4-7
33.25 %	Fifth scale degree Balinese pentatonic triads (0-5-11)	TSC3-5A
	Second scale degree Balinese pentatonic triads (0-6-11)	TSC3-5B
33.11 %	0-4-10-11 tetrachords (0-4-10-11)	TSC4-5A
	0-1-7-11 tetrachords (0-1-7-11)	TSC4-5B
33.02 %	0-1-5-6 tetrachords (0-1-5-6)	TSC4-8
32.83 %	Dream chords (0-1-2-7)	TSC4-6
	0-5-6-7 tetrachords (0-5-6-7)	
32.74 %	First inversion diminished M7 chords (0-3-6-11)	TSC4-18A
	0-5-8-11 tetrachords (0-5-8-11)	TSC4-18B
32.70 %	Diminished triads (0-3-6)	TSC3-10
32.50 %	Second inversion major-minor tetrachords (0-3-8-11)	TSC4-17
32.42 %	Messiaen's Truncated Mode 5 Inverse scale as chords (0-5-6-11)	TSC4-9
32.41 %	0-7-8-10 tetrachords (0-7-8-10)	TSC4-11A

	Warao tetratonic scale (Native South America) as chords (0-2-3-10)	TSC4-11B
32.33 %	French sixths (0-4-6-10)	TSC4-25
32.28 %	0-1-4-10 tetrachords (0-1-4-10)	TSC4-12A
	0-6-9-10 tetrachords (0-6-9-10)	TSC4-12B
32.09 %	Third inversion augmented sevenths (0-2-6-10)	TSC4-24
	Augmented sevenths (0-4-8-10)	
32.00 %	0-1-6-11 tetrachords (0-1-6-11)	TSC4-6
	0-5-10-11 tetrachords (0-5-10-11)	
31.72 %	All-interval tetrachords A (0-1-4-6)	TSC4-Z15A
	0-2-5-6 tetrachords (0-2-5-6)	TSC4-Z15B
31.67 %	0-1-4-5 tetrachords (0-1-4-5)	TSC4-7
31.59 %	Webern trichords (0-1-6)	TSC3-5A
	0-5-6 trichords (0-5-6)	TSC3-5B
31.43 %	Tritones (0-6)	TSC2-6
31.22 %	Second inversion Italian sixths (0-2-6)	TSC3-8A
	Major flat 5 trichords (0-4-6)	TSC3-8B
30.98 %	Second inversion augmented sevenths (0-2-4-8)	TSC4-24
	First inversion augmented sevenths (0-4-6-8)	
30.89 %	0-3-5-11 tetrachords (0-3-5-11)	TSC4-Z15A
	0-6-8-11 tetrachords (0-6-8-11)	TSC4-Z15B
30.61 %	0-1-3-6 tetrachords (0-1-3-6)	TSC4-13A
	0-3-5-6 tetrachords (0-3-5-6)	TSC4-13B
30.33 %	0-1-5-11 tetrachords (0-1-5-11)	TSC4-5A
	0-6-10-11 tetrachords (0-6-10-11)	TSC4-5B
	0-3-9-11 tetrachords (0-3-9-11)	TSC4-12A
	0-2-8-11 tetrachords (0-2-8-11)	TSC4-12B
30.18 %	Prime number seventh Bs (0-1-3-5)	TSC4-11A
	0-2-4-5 tetrachords (0-2-4-5)	TSC4-11B
30.05 %	0-5-6-8 tetrachords (0-5-6-8)	TSC4-Z29A
	0-2-3-8 tetrachords (0-2-3-8)	TSC4-Z29B
30.00 %	Minor sixths (0-8)	TSC2-4
	0-8-9 trichords (0-8-9)	TSC3-3A
	0-1-9 trichords (0-1-9)	TSC3-3B
29.82 %	0-1-9-10 tetrachords (0-1-9-10)	TSC4-3
29.68 %	0-2-4-6 tetrachords (0-2-4-6)	TSC4-21
29.63 %	0-9-10 trichords (0-9-10)	TSC3-2A
	0-1-10 trichords (0-1-10)	TSC3-2B
29.55 %	First inversion French sixths (0-2-6-8)	TSC4-25

29.50 %	0-6-7-8 tetrachords (0-6-7-8)	TSC4-5A
	0-1-2-8 tetrachords (0-1-2-8)	TSC4-5B
	0-6-8-9 tetrachords (0-6-8-9)	TSC4-12A
	0-1-3-9 tetrachords (0-1-3-9)	TSC4-12B
29.13 %	0-2-4-10 tetrachords (0-2-4-10)	TSC4-21
	0-6-8-10 tetrachords (0-6-8-10)	
29.07 %	0-2-11 trichords (0-2-11)	TSC3-2A
	0-9-11 trichords (0-9-11)	TSC3-2B
	Prime number seventh As (0-2-3-5)	TSC4-10
28.94 %	0-2-3-6 tetrachords (0-2-3-6)	TSC4-12A
	0-3-4-6 tetrachords (0-3-4-6)	TSC4-12B
28.89 %	Minor sevenths (0-10)	TSC2-2
	0-1-4 trichords (0-1-4)	TSC3-3A
	0-3-4 trichords (0-3-4)	TSC3-3B
28.39 %	0-1-2-6 tetrachords (0-1-2-6)	TSC4-5A
	0-4-5-6 tetrachords (0-4-5-6)	TSC4-5B
28.15 %	0-2-4 trichords (0-2-4)	TSC3-6
27.88 %	First inversion Italian sixths (0-6-8)	TSC3-8A
	First inversion major flat 5 trichords (0-2-8)	TSC3-8B
27.46 %	0-2-8-10 tetrachords (0-2-8-10)	TSC4-21
27.31 %	0-2-3-11 tetrachords (0-2-3-11)	TSC4-3
	0-8-9-11 tetrachords (0-8-9-11)	
27.22 %	0-3-11 trichords (0-3-11)	TSC3-3A
	0-8-11 trichords (0-8-11)	TSC3-3B
27.04 %	0-2-10 trichords (0-2-10)	TSC3-6
	0-8-10 trichords (0-8-10)	
26.57 %	0-2-10-11 tetrachords (0-2-10-11)	TSC4-2A
	0-1-9-11 tetrachords (0-1-9-11)	TSC4-2B
25.92 %	0-1-3-4 tetrachords (0-1-3-4)	TSC4-3
25.74 %	0-8-9-10 tetrachords (0-8-9-10)	TSC4-2A
	0-1-2-10 tetrachords (0-1-2-10)	TSC4-2B
24.91 %	0-1-10-11 tetrachords (0-1-10-11)	TSC4-1
24.35 %	0-1-3-11 tetrachords (0-1-3-11)	TSC4-2A
	0-8-10-11 tetrachords (0-8-10-11)	TSC4-2B
24.07 %	0-1-2-4 tetrachords (0-1-2-4)	TSC4-2A
	0-2-3-4 tetrachords (0-2-3-4)	TSC4-2B
23.80 %	0-1-2-11 tetrachords (0-1-2-11)	TSC4-1
	0-9-10-11 tetrachords (0-9-10-11)	
22.96 %	0-1-3 trichords (0-1-3)	TSC3-2A
	0-2-3 trichords (0-2-3)	TSC3-2B
22.22 %	Major seconds (0-2)	TSC2-2

20.18 %	0-1-11 trichords (0-1-11) 0-10-11 trichords (0-10-11)	TSC3-1
19.63 %	0-1-2-3 clusters (0-1-2-3)	TSC4-1
18.33 %	Major sevenths (0-11)	TSC2-1
16.29 %	Chromatic clusters (0-1-2)	TSC3-1
13.33 %	Minor seconds (0-1)	TSC2-1

Figure 3.1 The harmonic similarity of all single pitches, two-pitch chords, three-pitch chords, and four-pitch chords.

It is not the purpose of this book to impose rules or restrictions on composers or to promote or discourage any particular way of composing. On the contrary, the purpose is to open up new opportunities. For example, a composer may wish to consider whether it would be useful to set upper and lower limits for the harmonic similarity of an entire composition, or just for elements such as specific movements, instruments, articulations, dynamics, motifs, rhythms, emotional expressions, operatic characters, and so on.

The root clarity and root ambiguity of chords can be viewed and compared across all of the second graph layers in Appendix A. Like the first layers, collectively, all of these layers resemble the skyline of a city.

For convenience, Figure 3.2 shows the root clarity or root ambiguity of all single-pitch, two-pitch, three-pitch and four-pitch transpositional set classes in declining order. Amongst these chords, only the dominant seventh chords and major triads (in root position and in all of their inversions inversion) have greater root clarity than single pitches.

Just as a composer may wish to consider whether it would be useful to set upper and lower limits for harmonic similarity, the same approach can be applied to degrees of root clarity and root ambiguity.

Root clarity percentages	**Transpositional set classes**	**Examples**
209.84 %	TSC4-27B	Dominant sevenths (0-4-7-10)
175.00 %	TSC3-11B	Major triads (0-4-7)
100.00 %	**TSC1-1**	**Single pitches (0)**
97.06 %	TSC4-22A	Major add 2 tetrachords (0-2-4-7)
96.43 %	TSC4-Z29B	0-4-6-7 tetrachords (0-4-6-7)
94.29 %	TSC3-7B	0-7-10 trichords (0-7-10)
77.42 %	TSC4-18A	Diminished M7 chords (0-1-4-7)
77.39 %	TSC4-10	0-7-9-10 tetrachords (0-7-9-10)
76.79 %	TSC4-17	Major-minor tetrachords (0-3-4-7)
	TSC4-19B	Augmented major sevenths (0-4-8-11)
75.24 %	TSC3-8A	Italian sixths (0-4-10)

75.00 %	TSC2-5	Perfect fifths (0-7)
	TSC3-5A	Webern trichords (0-1-6)
	TSC3-5B	0-1-7 trichords (0-1-7)
66.86 %	TSC4-22B	Minor add 4 tetrachords (0-3-5-7)
64.52 %	TSC4-13B	Mistuned Historical Church Bell 7 as chords (0-1-7-10)
63.93 %	TSC4-4B	0-7-10-11 tetrachords (0-7-10-11)
	TSC4-Z15A	All-interval tetrachord As (0-1-4-6)
57.14 %	TSC4-20	Major seventh tetrachords (0-4-7-11)
55.56 %	TSC2-4	Major thirds (0-4)
	TSC3-3A	0-1-4 trichords (0-1-4)
	TSC3-3B	0-1-9 trichords (0-1-9)
55.00 %	TSC3-10	Diminished triads (0-3-6)
51.62 %	TSC4-21	0-2-4-6 tetrachord (0-2-4-6)
48.39 %	TSC4-12A	0-1-4-10 tetrachords (0-1-4-10)
39.29 %	TSC3-9	Suspended triads (0-5-7)
	TSC4-6	Dream chords (0-1-2-7)
	TSC4-14A	Minor add 2 tetrachords (0-2-3-7)
	TSC4-16A	Major seventh plus four tetrachords (0-4-6-11)
32.38 %	TSC4-27A	Half-diminished sevenths (0-3-6-10)
31.43 %	TSC4-5A	0-4-10-11 tetrachords (0-4-10-11)
	TSC4-Z15B	0-2-5-6 tetrachords (0-2-5-6)
	TSC4-Z29A	All-interval tetrachord Bs (0-1-3-7)
21.74 %	TSC3-7A	Ute Tritonic scale as trichords (0-3-10)
16.57 %	TSC4-11A	0-7-8-10 tetrachords (0-7-8-10)
12.90 %	TSC4-13A	0-6-7-9 tetrachords (0-6-7-9)
12.82 %	TSC4-14B	Major add 4 tetrachords (0-4-5-7)
12.50 %	TSC3-4A	0-1-5 trichords (0-1-5)
	TSC3-4B	0-1-8 trichords (0-1-8)
	TSC3-11A	Minor triads (0-3-7)
	TSC4-4A	0-7-8-9 tetrachords (0-7-8-9)
	TSC4-5B	0-1-7-11 tetrachords (0-1-7-11)
	TSC4-7	0-7-8-11 tetrachords (0-7-8-11)
	TSC4-18B	0-3-6-7 tetrachords (0-3-6-7)
	TSC4-19A	Minor-major sevenths (0-3-7-11)
11.11 %	TSC3-8B	Major flat 5 trichords (0-4-6)
	TSC4-3	0-1-3-4 tetrachords (0-1-3-4)
10.06 %	TSC3-6	0-2-4 trichords (0-2-4)
	TSC4-2A	0-1-2-4 tetrachords (0-1-2-4)
	TSC4-2B	0-2-3-4 tetrachords (0-2-3-4)
	TSC4-11B	0-2-4-5 tetrachords (0-2-4-5)
7.84 %	TSC4-26	Major triad add sixes (0-4-7-9)
6.67 %	TSC2-2	Minor sevenths (0-10)
	TSC3-1	Chromatic clusters (0-1-2)
	TSC3-2A	0-1-3 trichords (0-1-3)
	TSC3-2B	0-2-3 trichords (0-2-3)

5.98 %	TSC4-16B	0-2-6-7 tetrachords (0-2-6-7)
5.14 %	TSC4-24	Augmented sevenths (0-4-8-10)
4.62 %	TSC4-23	Dominant seventh suspended fourths (0-5-7-10)
0.36 %	TSC4-12B	0-3-4-6 tetrachords (0-3-4-6)
0 % – 2 equal roots	TSC2-1	Minor seconds (0-1)
	TSC2-3	Minor thirds (0-3)
	TSC2-6	Tritones (0-6)
0 % - 2 equal roots	TSC4-1	0-1-2-3 clusters (0-1-2-3)
	TSC4-8	0-1-7-8 tetrachords (0-1-7-8)
	TSC4-9	Messiaen's Truncated Mode 5 scale as chords (0-1-6-7)
	TSC4-25	French sixths (0-4-6-10)
0 % – 3 equal roots	TSC3-12	Augmented triads (0-4-8)
0 % - 4 equal roots	TSC4-28	Diminished sevenths (0-3-6-9)

Figure 3.2 The root clarity or root ambiguity of all single-pitch, two-pitch, three-pitch and four-pitch transpositional set classes.

In practice, the suitability of a root candidate may vary by the octave in which it is placed beneath a chord. However, there is no requirement to place a root candidate beneath a chord – consider any octave.

3.3 Spectral-pitch and root-candidate substitutions within the same cardinality

Below, Figure 3.3 to Figure 3.6 show how these substitutions work using major triads in root position (0-4-7) as the starting point. They also show the extent to which the harmonic similarity and root clarity of the first and second chord are different.

There are only three close one-pitch substitutions from tonic major triads in root position (0-4-7) to other three-pitch chords, that retain the same degree of harmonic similarity as the tonic (Figure 3.3 and Figure 3.4). The first is spectral-pitch movement from tonic major triads to mediant minor triads in root position (I → iii); the second is root-candidate movement from tonic major triads to submediant first inversion minor triads (I → vi); and the third is root-candidate movement from tonic major triads to 0-7-9 trichords.

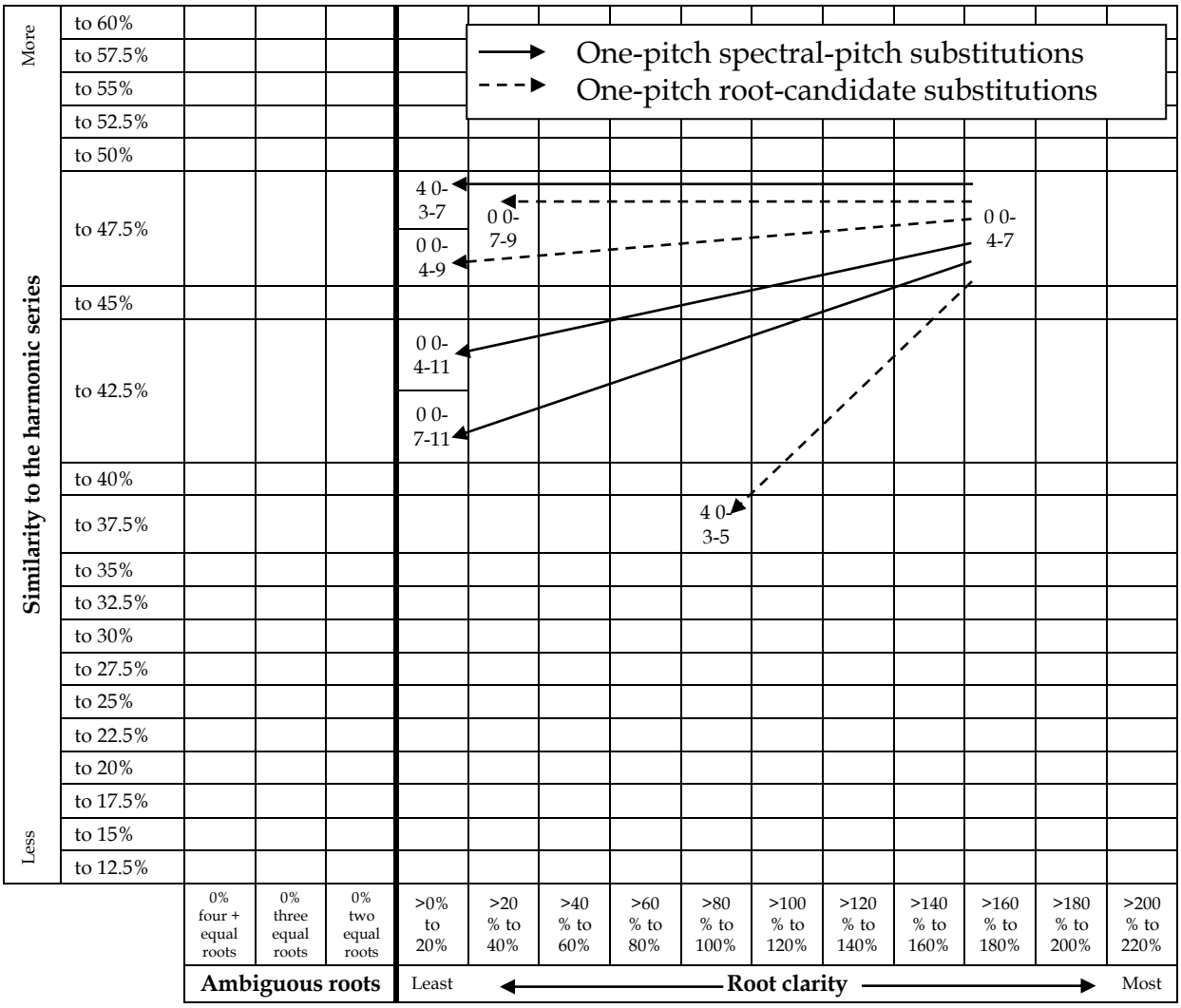


Figure 3.3 One-pitch spectral-pitch and root-candidate substitutions from major triads in root position (0-4-7) to other three-pitch chords.

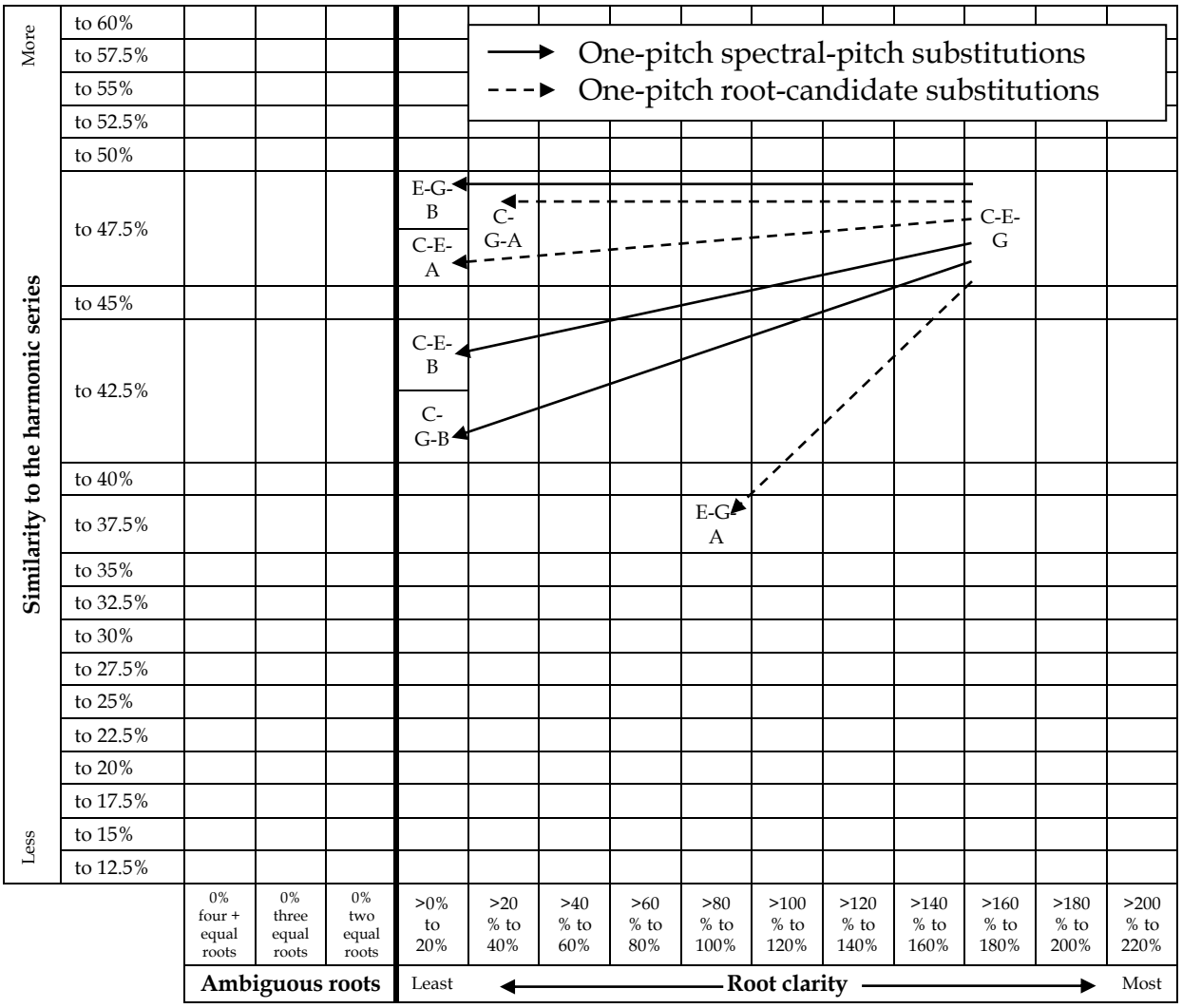


Figure 3.4 Letter names for one-pitch spectral-pitch and root-candidate substitutions from C major triads in root position (C-E-G) to other three-pitch chords.

There are only two close two-pitch substitutions from tonic major triads in root position (0-4-7) to other three-pitch chords, that retain the same degree of harmonic similarity and root clarity as the tonic (Figure 3.5 and Figure 3.6). The first is spectral-pitch movement from the tonic to the dominant major triads (I → V), and the second is root-candidate movement from the tonic to the subdominant major triads (I → IV).

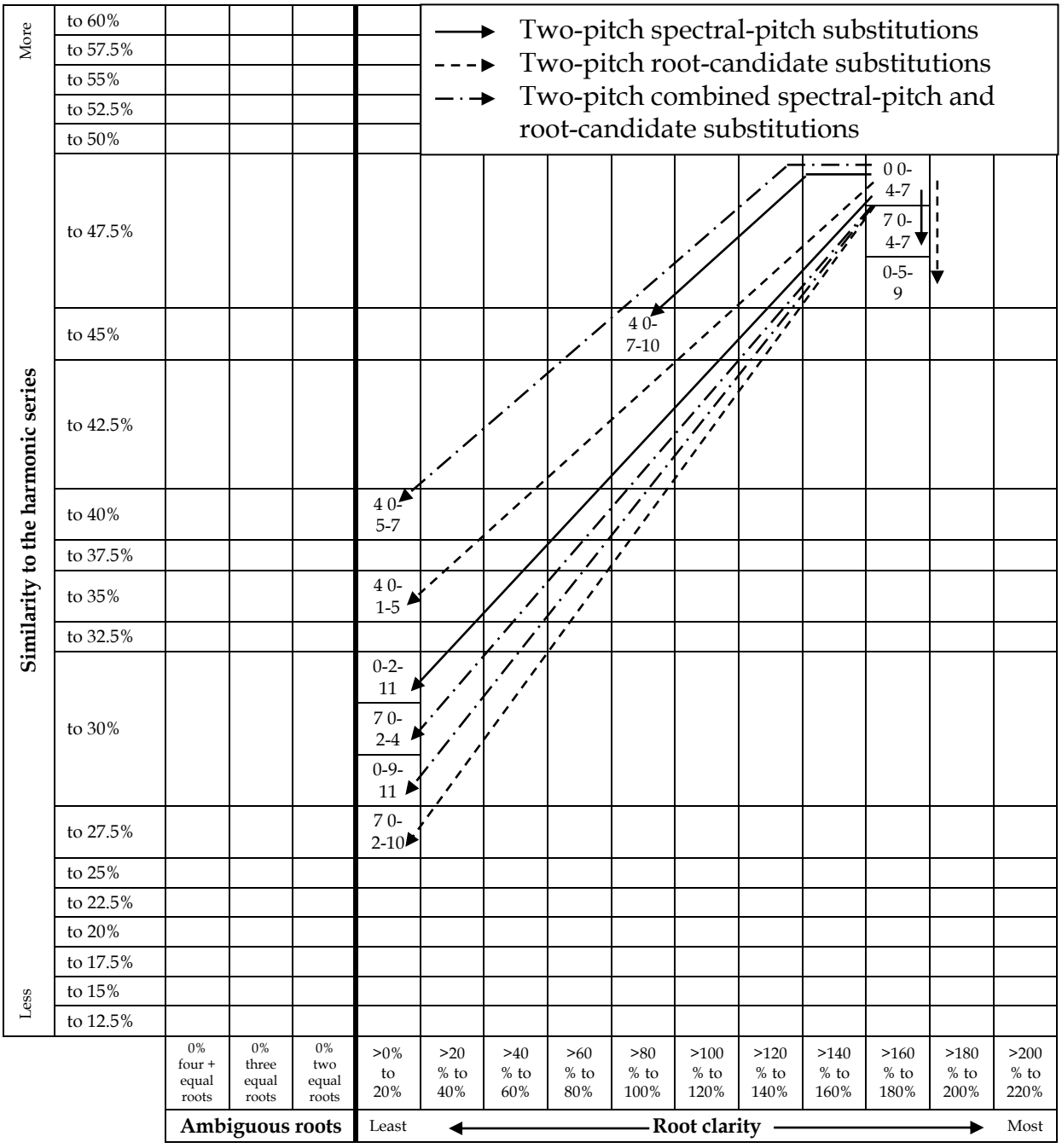


Figure 3.5 Two-pitch spectral-pitch and root-candidate substitutions from major triads in root position (0-4-7) to other three-pitch chords.

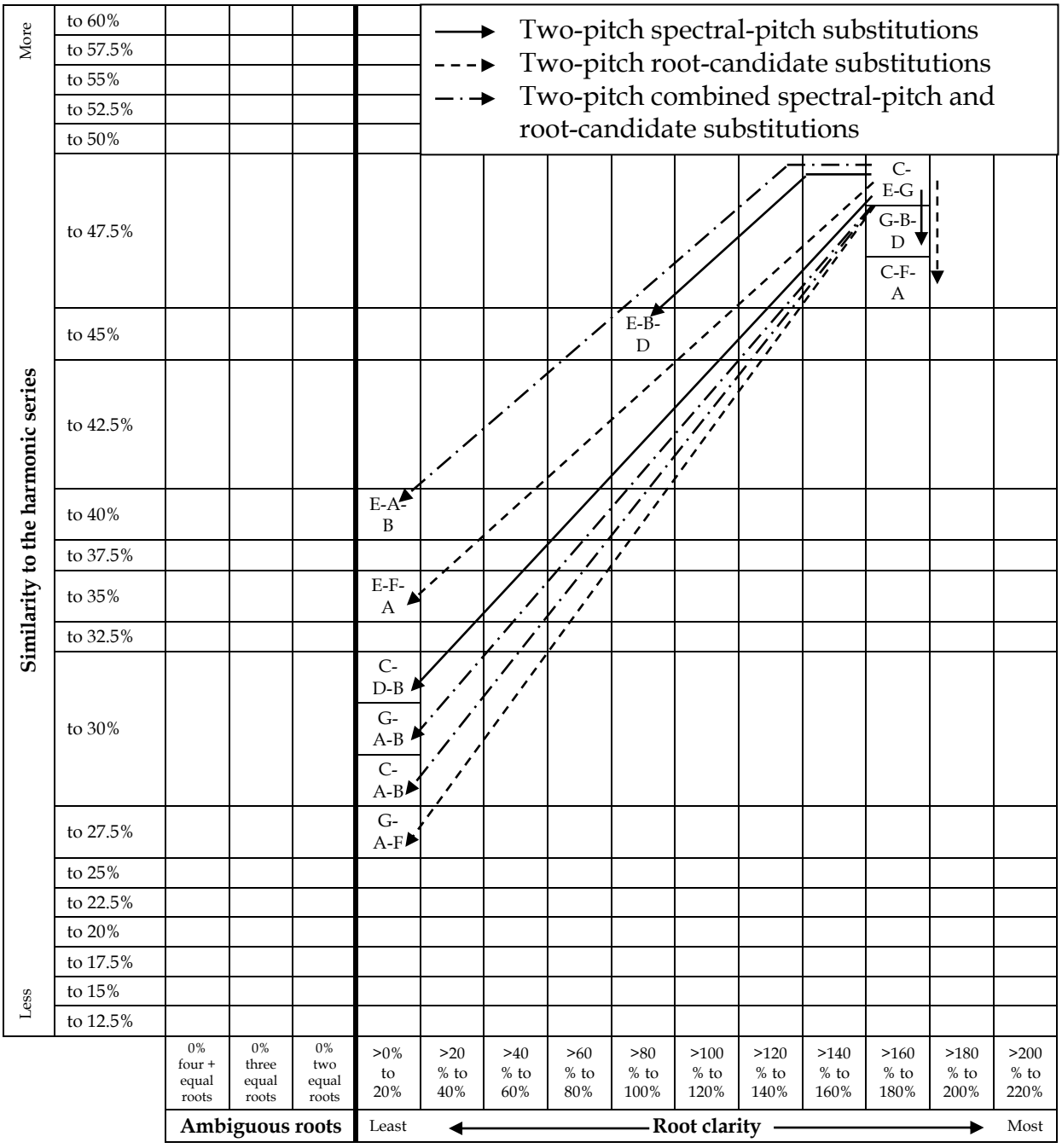


Figure 3.6 Letter names for two-pitch spectral-pitch and root-candidate substitutions from C major triads in root position (C-E-G) to other three-pitch chords.

3.4 Changing the cardinality of chords

The cardinality of a chord can be increased by adding one or more of its spectral pitches to the performed pitches in the chord. The spectral pitches at the top of the series should fit better with the performed pitches of the chord than the spectral pitches at the bottom

of the series, with the qualification that spectral pitches with the same weight should fit equally well.

Likewise, the cardinality of a chord can be increased by adding one or more of its root candidates to the performed pitches in the chord. The root candidates at the top of the series should fit better with the performed pitches of the chord than the root candidates at the bottom of the series, with the qualification that root candidates with the same weight should fit equally well.

The cardinality of a chord can also be increased by adding a combination of its spectral pitches and root candidates.

In each instance, the similarity of the new chord to the harmonic series and the clarity of its root can vary.

In Figure 3.1 and Figure 3.2, the single pitch serves as a useful benchmark, as it reflects a single human voice when speaking or singing.

Figure 3.7 shows the spectral-pitch pathway from single pitches. Each of the chords in the pathway divides the octave nearly evenly (compare Figure 2.3 with Figure 3.7).

Figure 3.8 shows the sequential root-candidate pathway from single pitches. Each of the chords in this pathway also divides the octave nearly evenly (compare Figure 2.3 with Figure 3.8). Since virtual roots are heard in an octave below the performed chordal pitches, this chart places the root of each chord below the previous chord in the pathway. This approach moves us away from transpositional twelve-point circular space into a descending transpositional twelve-point circular spiral, to some degree. It is a bridge between the transpositional twelve-point circular space used in our method and the more complex psychoacoustics spaces in which we hear performed chordal pitches.

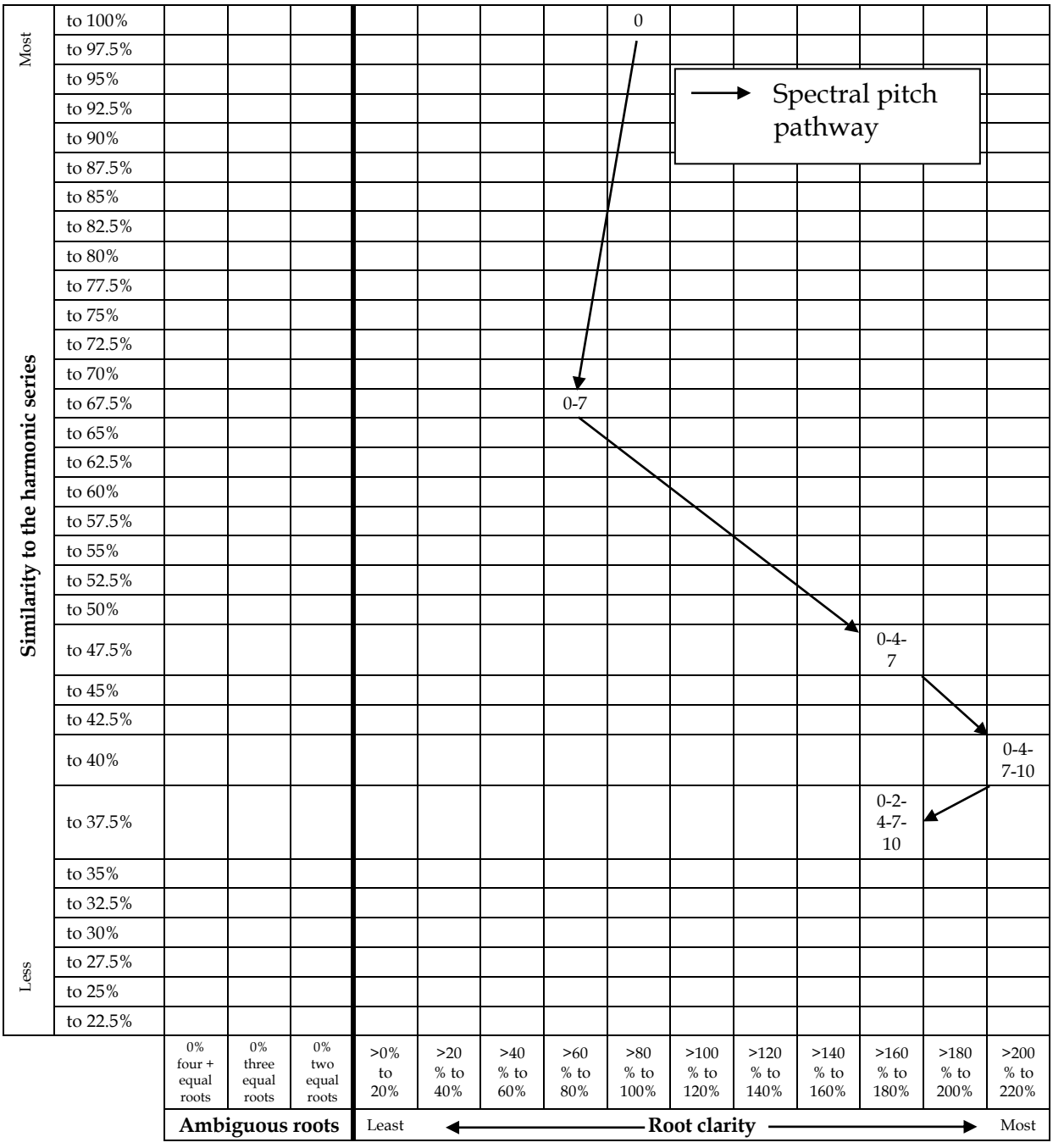


Figure 3.7 The spectral-pitch pathway from single pitches.

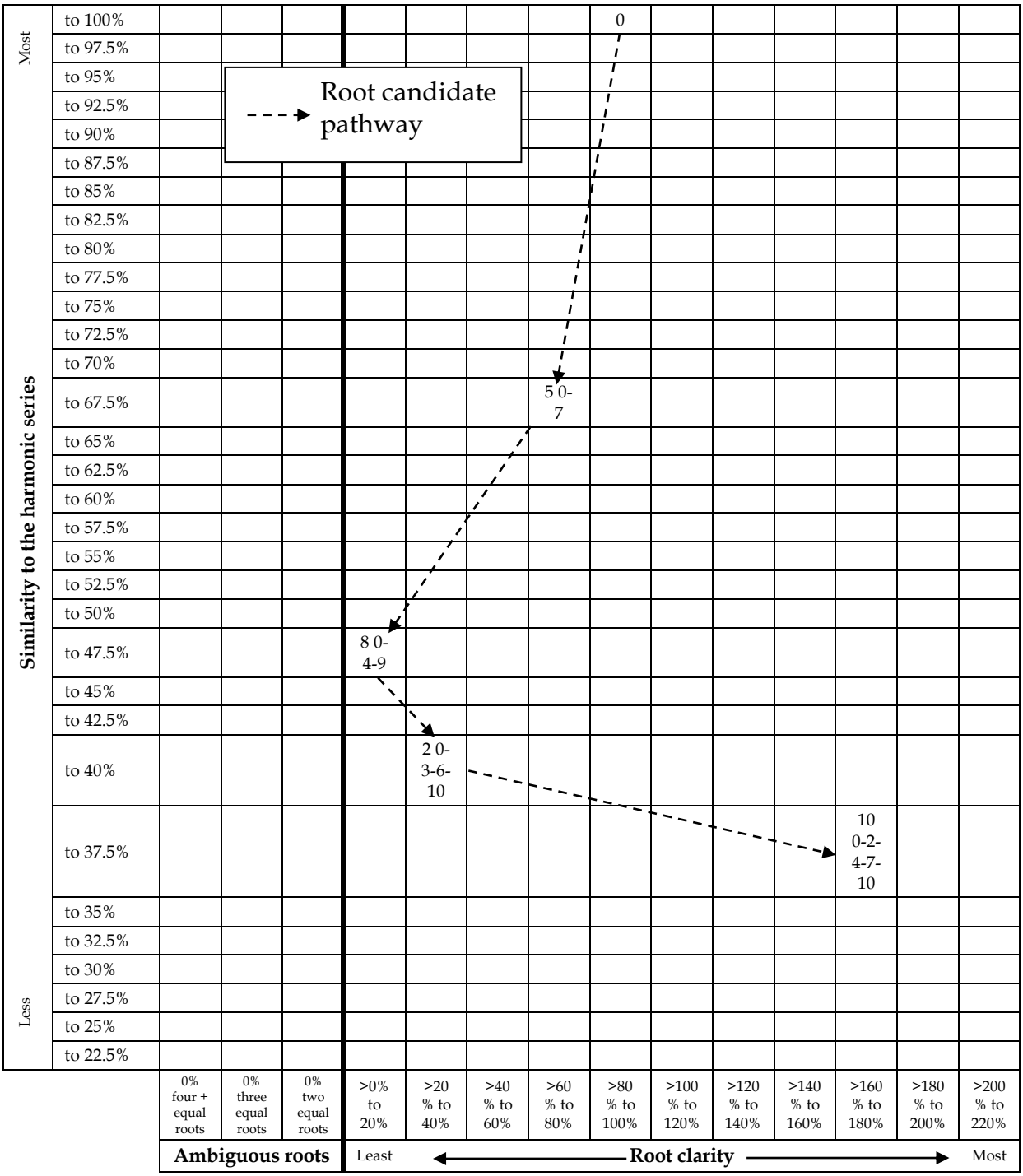


Figure 3.8 The root-candidate pathway from single pitches.

Figure 3.9 shows the spectral-pitch pathways from major triads in root position (0-4-7). There are alternative routes where two spectral pitches have the same weight. Major triads in root position also appear in Figure 3.7. The difference between these two charts is that Figure 3.7 shows a spectral pathway that is based on the spectral pitches of single pitches, while Figure 3.9 shows the spectral pathways of major triads in root position.

They are distinct pathways that lead in different directions. The chords in the spectral-pitch pathways from major triads do not divide the octave nearly evenly.

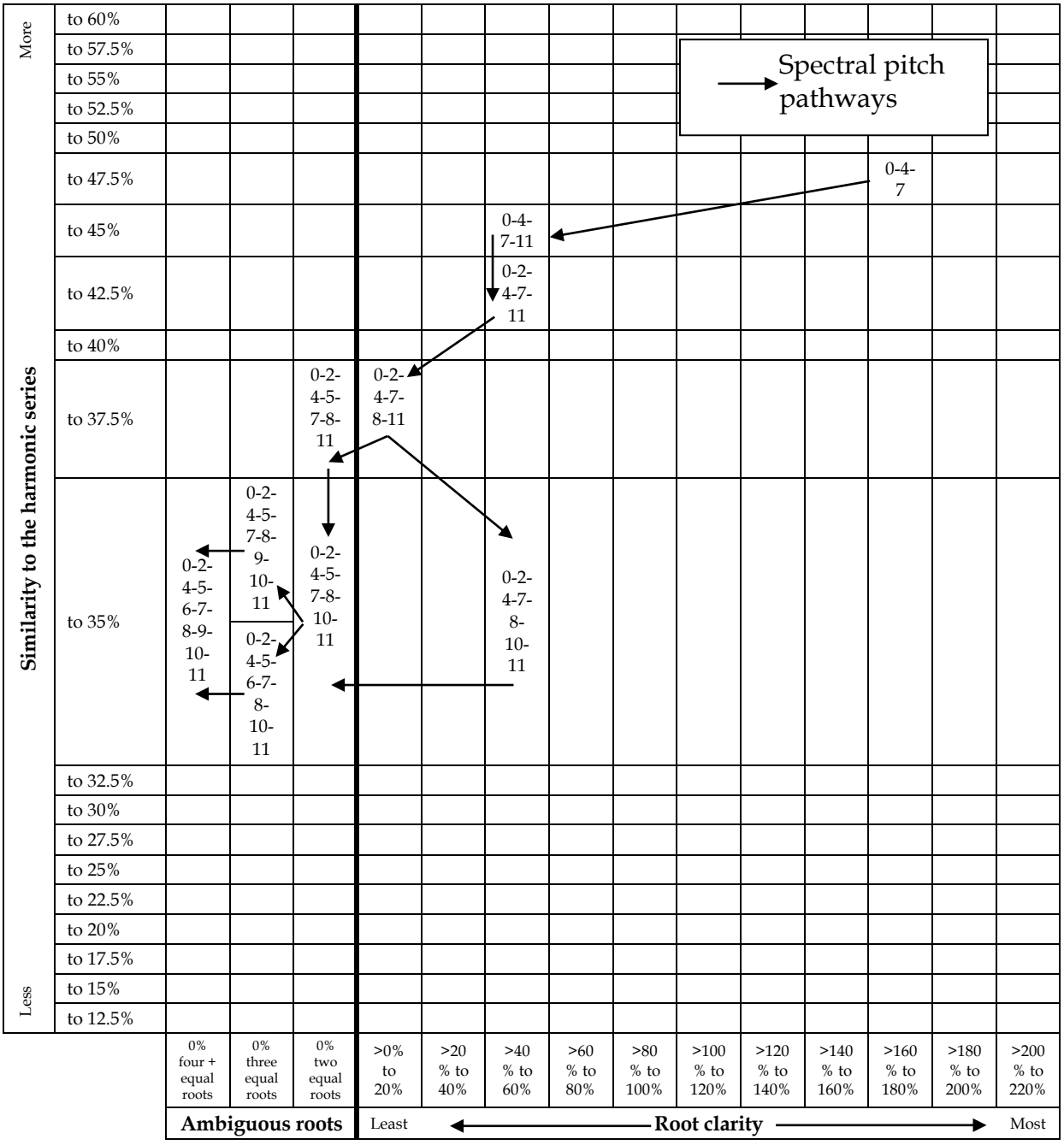


Figure 3.9 The spectral-pitch pathways from major triads in root position (0-4-7).

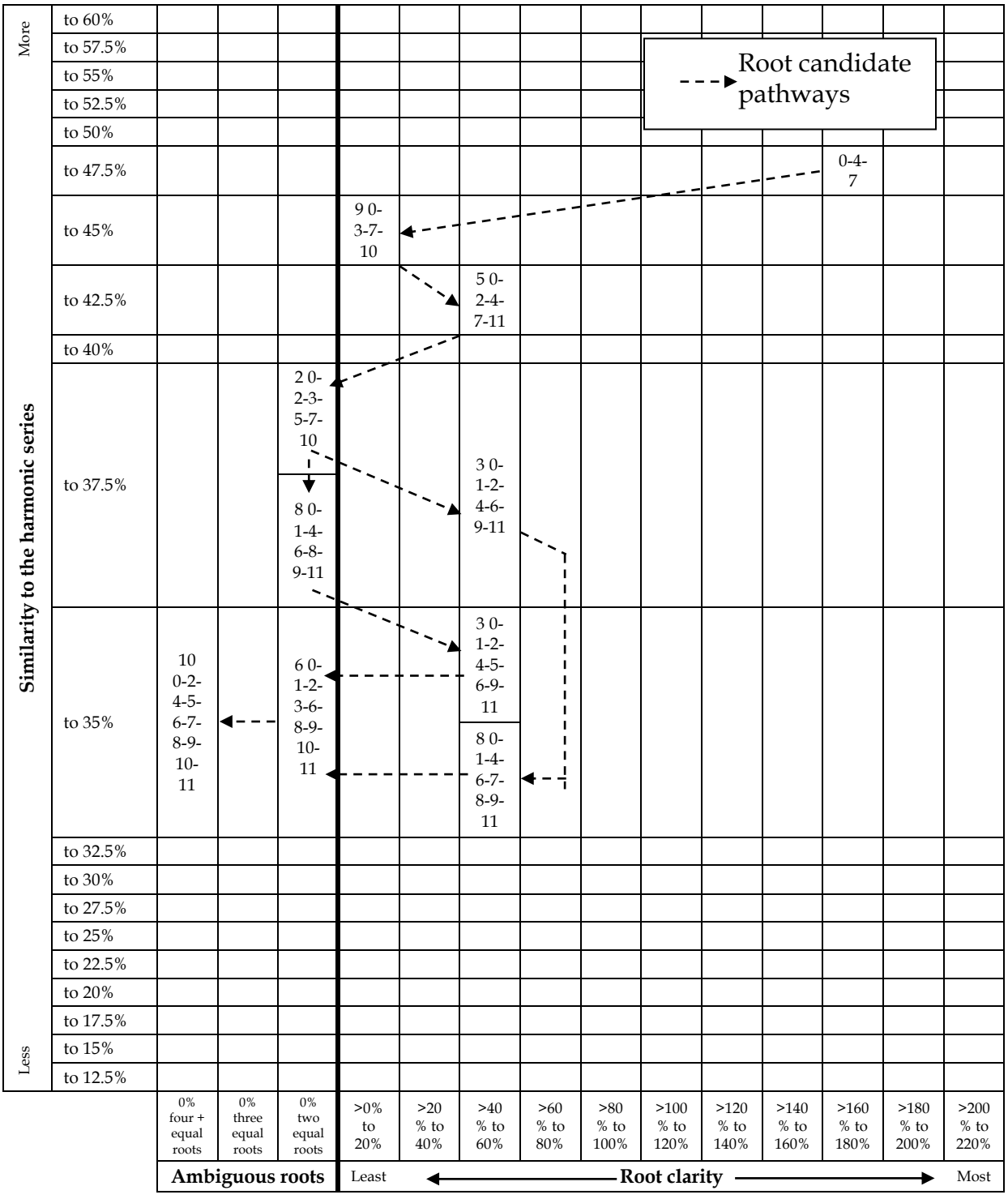


Figure 3.10 The root-candidate pathways from major triads in root position (0-4-7).

3.4 Conclusion

In my view, the result of this multi-layered approach is a unique and practical method that will allow composers to accurately evaluate the sound of any chord, and compare the sound of different chords. The method is easy to use but is not too long; it retains the key elements of the sound of chords at the sensory level. It also provides a springboard for composers to make more informed decisions about the implementation of chords in a musical composition by using different instruments, timbres, dynamics, articulations, registers and frequencies.

4
The Foundation for Harmony and Tonality

In this chapter, we extend our analysis briefly to chords performed in succession over time.

4.1 Basic chord progressions in the major scale

Consider the movement from a tonic major triad in root position to a dominant major triad in root position (I – V), one of the most basic chord progressions in tonal music. Assume that the chord progression occurs without any rest between the chords (Figure 4.1). The dominant triad retains one pitch from the tonic triad but adds two new pitches, which are the two non-chordal pitches that are closest to the tonic triad by spectral weight (Figure 4.2).

0 0-4-7	→	7 0-4-7					
		0 7-11-2					Spectral chord progression
I	→	V					

Figure 4.1 I – V spectral chord progression.

0-4-7 Spectral pitches and weights									
7	4	0	**11**	**2**	8	5	10	6	9
150.00	133.33	100.00	**83.33**	**82.50**	33.33	20.00	20.00	12.50	12.50

Figure 4.2 Spectral pitches and weights for a major triad in root position (0-4-7).

Next, consider the movement from a tonic major triad in root position to a subdominant major triad in root position (I – IV), another basic chord progression in tonal music. Again, assume that the chord progression occurs without any rest between the chords (Figure 4.3). The subdominant triad retains one pitch from the tonic triad but adds two new pitches, which are the two non-chordal pitches that are the closest root candidates to the tonic triad by weight (Figure 4.4).

0 0-4-7	→	5 0-4-7					
		0 5-9-0					Root chord progression
I	→	IV					

Figure 4.3 I – IV root chord progression.

0-4-7 Root candidates and weights									
0	4	7	**9**	**5**	2	3	8	6	10
275.00	100.00	100.00	**81.67**	**72.92**	37.92	33.33	33.33	20.00	12.50

Figure 4.4 Root candidates and weights for a major triad in root position (0-4-7).

It is clear that the I – V and I – IV chord progressions reflect the psychoacoustic processing of spectral pitches and root candidates in the brain. The weights of the spectral pitches or root candidates, respectively, which are added as fundamental pitches in the second chord, are higher than the weights of the spectral pitches and root candidates of all other pitches (including those not shown in the spectral pitch or root candidate series).

These chord progressions sound good for two reasons. First, there is harmonic consistency between the first and second chords in the sense that the fundamental pitches are structurally similar to one another (both are major triads) [27]. Second, there is harmonic consistency between the first and second chords in the sense that the second chord is closely related to the first chord by minimal psychoacoustic distance, measured by the weights of the spectral pitches and root candidates of the first chord.

Now, consider the common I-IV-V-I chord progression. Again, we will use triads in root position. (Although this analysis may also be performed with inversions, there may be changes to root clarity which would need to be taken into account). Assume that each chord progression occurs without any rest between the chords.

First, consider the progression of each chord to the next chord, as pairs. The I – IV and V – I changes are root chord progressions that each adds pitches that are the closest root candidates of the preceding chord.

0 0-4-7	→	0 5-9-0					Root chord progression
		5 0-4-7	→	5 2-6-9			Mixed spectral (2, 6) and root (9, 6) progression. Pitch 6 is a weak fit. As pitch 6 is the leading note (11) of the 0 major scale, this weakness might underlie its "desire" to be resolved by a stronger tonic pitch.
				7 0-4-7	→	7 5-9-0	Root chord progression
						0 0-4-7	
I	→	IV	→	V	→	I	Complete chord progression

Figure 4.5

Above, the IV – V change adds pitches that are either spectral pitches or root candidates, which are set out in bold below.

0-4-7 Spectral pitches and weights									
7	4	0	11	**2**	8	5	10	**6**	**9**
150.00	133.33	100.00	83.33	**82.50**	33.33	20.00	20.00	**12.50**	**12.50**

Figure 4.6

0-4-7 Root candidates and weights									
0	4	7	**9**	5	**2**	3	8	**6**	10
275.00	100.00	100.00	**81.67**	72.92	**37.92**	33.33	33.33	**20.00**	12.50

Figure 4.7

Second, consider the I-IV-V-I chord progression in terms of how each chord relates to the original tonic chord.

0 0-4-7	→	0 5-9-0					Root chord progression
		0 5-9-0	→	0 7-11-2			Spectral chord progression
				0 7-11-2	→	0 0-4-7	Return to the fundamental pitches of the tonic chord.
I	→	IV	→	V	→	I	Complete chord progression

Figure 4.8

When the tonic environment is considered, it is clear that the I-IV-V-I chord progression is based on repeated tonic pitches together with spectral pitches and root candidates that are closest to the tonic triad by weight.

0-4-7 Spectral pitches and weights									
7	**4**	**0**	**11**	**2**	8	5	10	6	9
150.00	**133.33**	**100.00**	**83.33**	**82.50**	33.33	20.00	20.00	12.50	12.50

Figure 4.9

0-4-7 Root candidates and weights									
0	**4**	**7**	**9**	**5**	2	3	8	6	10
275.00	**100.00**	**100.00**	**81.67**	**72.92**	37.92	33.33	33.33	20.00	12.50

Figure 4.10

Again, it is clear that the psychoacoustic processing in the brain – of the spectral pitches and root candidates of the tonic chord – is deeply embedded in the I-IV-V-I chord progression. This chord progression sounds good because there is harmonic consistency between the subdominant and dominant chords and final tonic chord in terms of structural similarity to, and minimal psychoacoustic distance from, the original tonic chord.

WHAT ABOUT NATURAL MINOR? (0-2-3-5-7-8-10) i-iv-v-i

4.2 Basic chord progressions in the natural minor scale

0-3-7 Spectral pitches and weights

7	0	3	**10**	**2**	4	11	5	1	9
183.33	100.00	100.00	**70.00**	**62.50**	33.33	33.33	32.50	20.00	12.50

Figure 4.11

0-3-7 Root candidates and weights

0	3	**5**	7	**8**	11	2	9	1	10
175.00	155.56	**123.75**	100.00	**97.22**	33.33	20.00	20.00	12.50	12.50

Figure 4.12

0 0-3-7	→	0 7-10-2					Spectral chord progression
i	→	v					

Figure 4.13

0 0-3-7	→	0 5-8-0					Root chord progression
i	→	iv					

Figure 4.14

Analysis based on each individual chord of the i-iv-v-i chord progression shows that it can be played at moderate or slow speed and the chords will sound like they belong.

0 0-3-7	→	0 5-8-0					Root chord progression
		5 0-3-7	→	5 2-5-9			Mixed spectral (2, 9) and root (5, 9) progression. Pitch 9 is a weak fit.
				7 0-3-7	→	7 5-8-0	Root chord progression
						0 0-3-7	
i	→	iv	→	v	→	i	Complete chord progression

Figure 4.15

Analysis based on the spectral pitches and root candidates of the original tonic chord shows that the i-iv-v-i chord progression can be played at high speed and the chords will also sound like they belong.

0 0-3-7	→	0 5-8-0					Root chord progression
		0 5-8-0	→	0 7-10-2			Spectral chord progression
				0 7-10-2	→	0 0-3-7	Return to the fundamental pitches of the tonic chord.
i	→	iv	→	v	→	i	Complete chord progression

Figure 4.16

Appendix A

The Method for Evaluating and Comparing the Sound of Chords

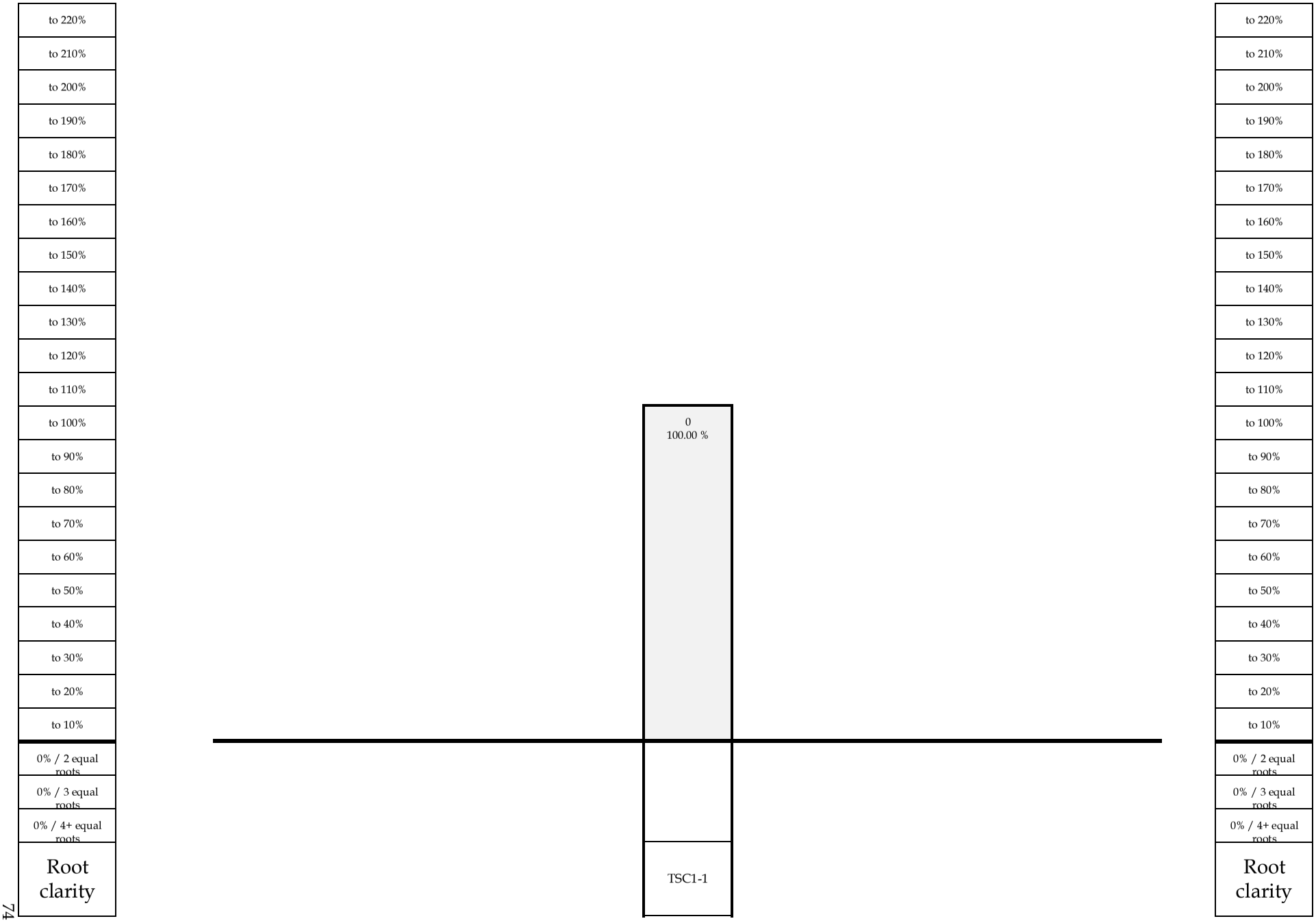
Single pitches

The similarity of chords to the harmonic series is presented on a scale from 0.00% to 70.00% in this Appendix. However, as the similarity of single pitches to the harmonic series extends beyond the scale used, it is simply noted below.

The similarity of single pitches [0] to the harmonic series is:

100.00 %

0 0 [100.00] 7 [50.00] 4 [33.33] 10 [20.00] 2 [12.50]	TSC1-1


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0
100.00 %
TSC1-1
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

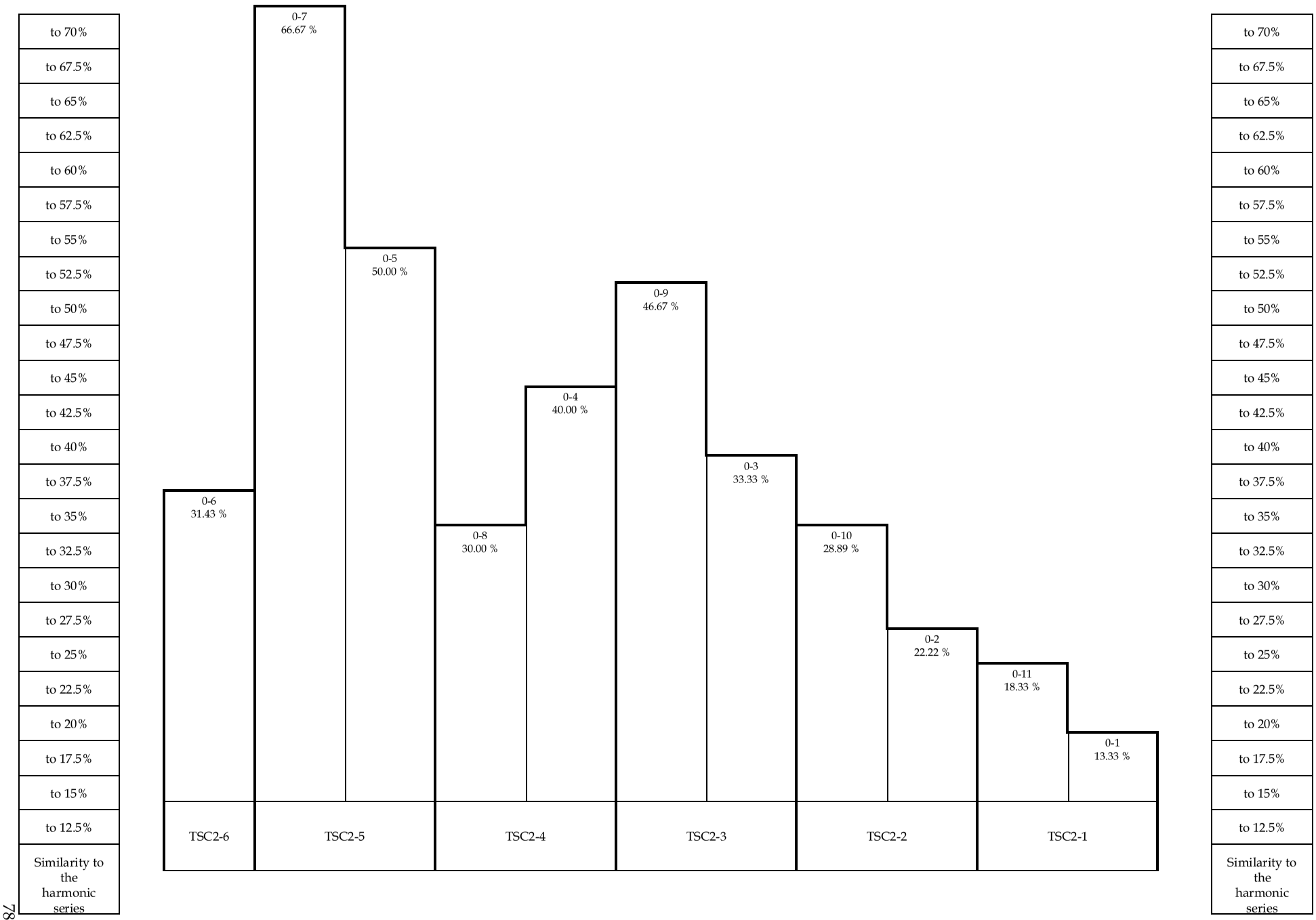
Root candidates and weights

Names

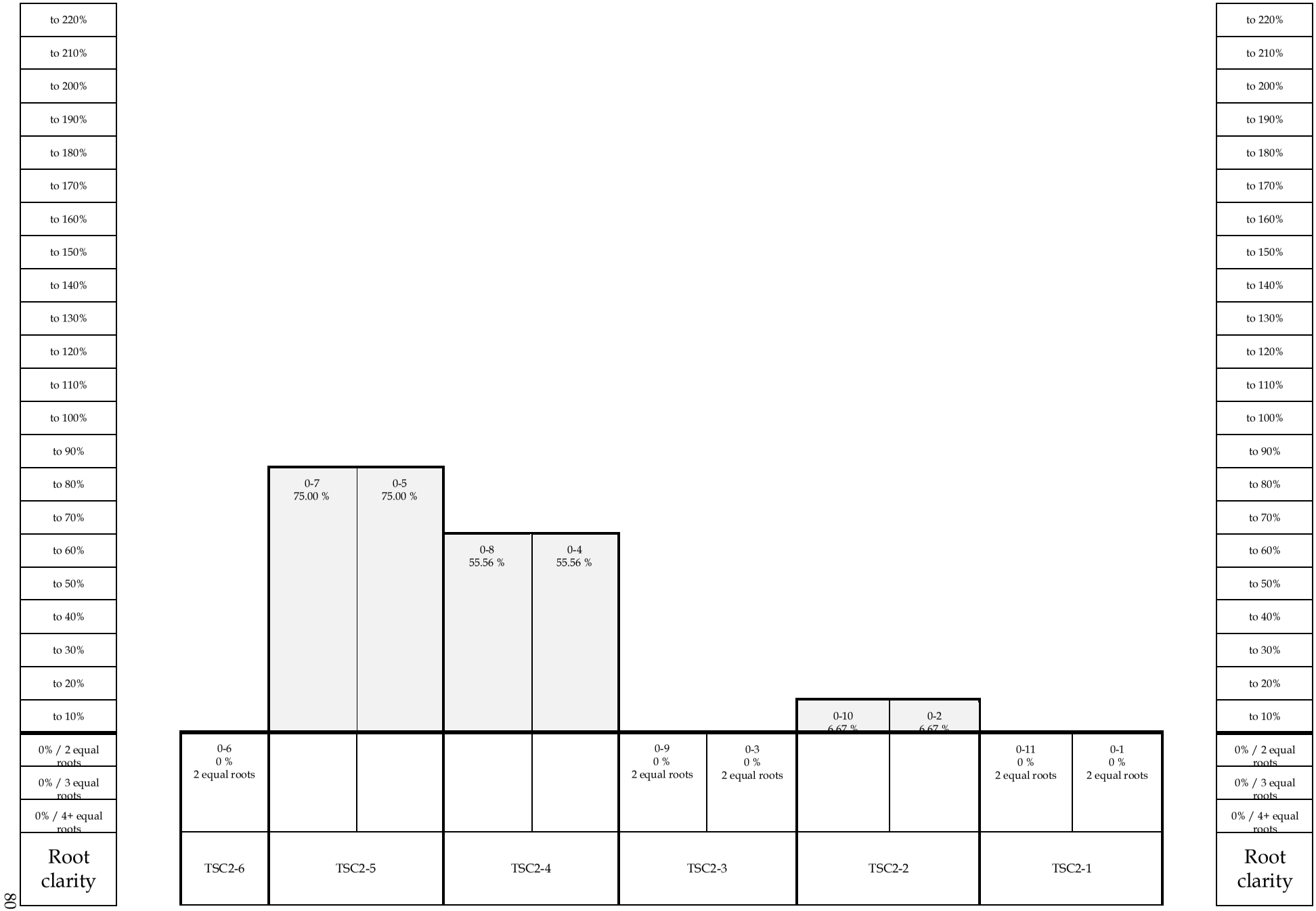
0 Single pitch Single note Unison Perfect unison Shadja (Northern Indian (Hindustani) classical music)	TSC1-1

Names

Two-pitch chords (intervals)


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-6
31.43 %
0-7
66.67 %
0-5
50.00 %
0-8
30.00 %
0-4
40.00 %
0-9
46.67 %
0-3
33.33 %
0-10
28.89 %
0-2
22.22 %
0-11
18.33 %
0-1
13.33 %
TSC2-6
TSC2-5
TSC2-4
TSC2-3
TSC2-2
TSC2-1
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC2-6	TSC2-5	TSC2-4	TSC2-3	TSC2-2	TSC2-1
0-6	0-7	0-8	0-9	0-10	0-11	0-2	0-3	0-4	0-5	0-1
0 [100.00]	7 [150.00]	0 [133.33]	0 [100.00]	10 [120.00]	0 [100.00]	0 [120.00]	0 [100.00]	4 [133.33]	0 [150.00]	0 [100.00]
6 [100.00]	0 [100.00]	8 [100.00]	9 [100.00]	0 [112.50]	11 [100.00]	2 [112.50]	3 [100.00]	0 [100.00]	5 [100.00]	1 [100.00]
4 [53.33]	2 [62.50]	3 [50.00]	4 [83.33]	5 [50.00]	6 [50.00]	7 [50.00]	7 [83.33]	7 [50.00]	7 [62.50]	7 [50.00]
10 [53.33]	4 [33.33]	7 [50.00]	7 [70.00]	7 [50.00]	7 [50.00]	9 [50.00]	10 [70.00]	11 [50.00]	4 [33.33]	8 [50.00]
1 [50.00]	11 [33.33]	4 [33.33]	1 [33.33]	2 [45.83]	3 [33.33]	4 [45.83]	4 [33.33]	8 [33.33]	9 [33.33]	4 [33.33]
7 [50.00]	5 [20.00]	10 [32.50]	10 [20.00]	4 [33.33]	4 [33.33]	6 [33.33]	1 [20.00]	2 [32.50]	3 [20.00]	5 [33.33]
2 [12.50]	10 [20.00]	6 [20.00]	2 [12.50]	8 [20.00]	9 [20.00]	10 [20.00]	2 [12.50]	10 [20.00]	10 [20.00]	10 [20.00]
8 [12.50]	9 [12.50]	2 [12.50]	11 [12.50]		10 [20.00]		5 [12.50]	6 [12.50]	2 [12.50]	11 [20.00]
					1 [12.50]					2 [12.50]
					2 [12.50]					3 [12.50]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to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-6
0 %
2 equal roots
0-7
75.00 %
0-5
75.00 %
0-8
55.56 %
0-4
55.56 %
0-9
0 %
2 equal roots
0-3
0 %
2 equal roots
0-10
6.67 %
0-2
6.67 %
0-11
0 %
2 equal roots
0-1
0 %
2 equal roots
TSC2-6
TSC2-5
TSC2-4
TSC2-3
TSC2-2
TSC2-1

TSC2-1	TSC2-2	TSC2-3	TSC2-4	TSC2-5	TSC2-6
0-1	0-2	0-3	0-4	0-5	0-6
0 [100.00]	2 [140.00]	0 [100.00]	0 [155.56]	5 [175.00]	0 [100.00]
1 [100.00]	0 [131.25]	3 [100.00]	4 [100.00]	0 [100.00]	6 [100.00]
5 [50.00]	10 [53.47]	8 [97.22]	5 [50.00]	10 [72.92]	2 [62.22]
6 [50.00]	5 [50.00]	5 [81.67]	9 [50.00]	1 [33.33]	8 [62.22]
8 [33.33]	7 [50.00]	11 [33.33]	2 [37.92]	8 [33.33]	5 [50.00]
9 [33.33]	8 [33.33]	2 [20.00]	8 [33.33]	2 [20.00]	11 [50.00]
2 [20.00]	4 [20.00]	1 [12.50]	6 [20.00]	7 [20.00]	4 [12.50]
3 [20.00]		10 [12.50]	10 [12.50]	3 [12.50]	10 [12.50]
10 [12.50]					
11 [12.50]					

TSC2-1	TSC2-2	TSC2-3	TSC2-4	TSC2-5	TSC2-6
0-11	0-10	0-9	0-8	0-7	
0 [100.00]	0 [140.00]	0 [100.00]	8 [155.56]	0 [175.00]	
11 [100.00]	10 [131.25]	9 [100.00]	0 [100.00]	7 [100.00]	
4 [50.00]	8 [53.47]	5 [97.22]	1 [50.00]	5 [72.92]	
5 [50.00]	3 [50.00]	2 [81.67]	5 [50.00]	3 [33.33]	
7 [33.33]	5 [50.00]	8 [33.33]	10 [37.92]	8 [33.33]	
8 [33.33]	6 [33.33]	11 [20.00]	4 [33.33]	2 [20.00]	
1 [20.00]	2 [20.00]	7 [12.50]	2 [20.00]	9 [20.00]	
2 [20.00]		10 [12.50]	6 [12.50]	10 [12.50]	
9 [12.50]					
10 [12.50]					

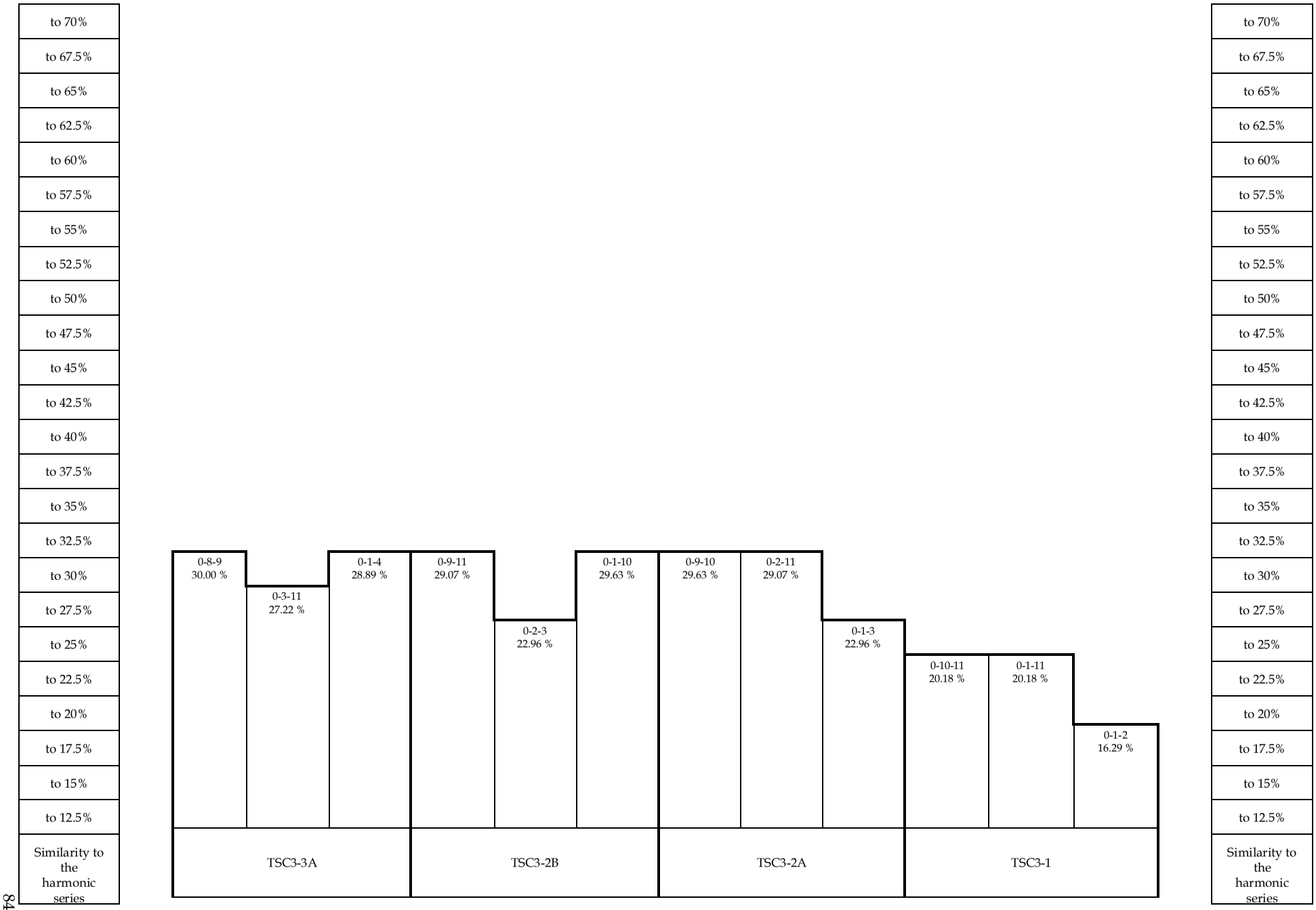
Root candidates and weights

Names

0-6	0-7	0-5	0-8	0-4	0-9	0-3	0-10	0-2	0-11	0-1
Tritone Augmented fourth Diminished fifth Tivra Madhyama (Northern Indian (Hindustani) classical music)	Perfect fifth Niagari (2-note scale) (Japan) Panchama (Northern Indian (Hindustani) classical music)	Perfect fourth Honchoshi (2-note scale) (Japan) Madhyama (Northern Indian (Hindustani) classical music)	Minor sixth Augmented fifth Komal Dhaivata (Northern Indian (Hindustani) classical music)	Major third Diminished fourth Shuddha Gandhara (Northern Indian (Hindustani) classical music)	Major sixth Shuddha Dhaivata (Northern Indian (Hindustani) classical music)	Minor third Komal Gandhara (Northern Indian (Hindustani) classical music)	Minor seventh Komal Nishada (Northern Indian (Hindustani) classical music) Warao Ditonic scale (Native South America)	Major second Shuddha Rhishabha (Northern Indian (Hindustani) classical music) Whole tone Whole step	Major seventh Shuddha Nishada (Northern Indian (Hindustani) classical music)	Minor second Komal Rhishabha (Northern Indian (Hindustani) classical music) Semitone Half step
TSC2-6	TSC2-5		TSC2-4		TSC2-3		TSC2-2		TSC2-1	

Names

Three-pitch chords
(triads and trichords)


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-8-9
30.00 %
0-3-11
27.22 %
0-1-4
28.89 %
0-9-11
29.07 %
0-2-3
22.96 %
0-1-10
29.63 %
0-9-10
29.63 %
0-2-11
29.07 %
0-1-3
22.96 %
0-10-11
20.18 %
0-1-11
20.18 %
0-1-2
16.29 %
TSC3-3A
TSC3-2B
TSC3-2A
TSC3-1
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC3-3A			TSC3-2B			TSC3-2A			TSC3-1		
0-8-9	0-3-11	0-1-4	0-9-11	0-2-3	0-1-10	0-9-10	0-2-11	0-1-3	0-10-11	0-1-11	0-1-2
0 [133.33]	3 [133.33]	4 [133.33]	9 [120.00]	0 [120.00]	10 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]	10 [120.00]	11 [120.00]	0 [120.00]
8 [100.00]	0 [100.00]	0 [100.00]	11 [112.50]	2 [112.50]	0 [112.50]	0 [112.50]	2 [112.50]	3 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]
9 [100.00]	11 [100.00]	1 [100.00]	0 [100.00]	3 [100.00]	1 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]
4 [83.33]	7 [83.33]	8 [83.33]	4 [83.33]	7 [83.33]	5 [83.33]	4 [83.33]	6 [83.33]	7 [83.33]	5 [50.00]	6 [50.00]	7 [50.00]
7 [70.00]	10 [70.00]	11 [70.00]	7 [70.00]	10 [70.00]	8 [70.00]	7 [70.00]	9 [70.00]	10 [70.00]	6 [50.00]	7 [50.00]	8 [50.00]
3 [50.00]	6 [50.00]	7 [50.00]	6 [50.00]	9 [50.00]	7 [50.00]	5 [50.00]	7 [50.00]	8 [50.00]	7 [50.00]	8 [50.00]	9 [50.00]
1 [33.33]	4 [33.33]	5 [33.33]	1 [45.83]	4 [45.83]	2 [45.83]	2 [45.83]	4 [45.83]	5 [45.83]	2 [45.83]	3 [45.83]	4 [45.83]
10 [32.50]	1 [32.50]	2 [32.50]	3 [33.33]	6 [33.33]	4 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	3 [33.33]	4 [33.33]	5 [33.33]
6 [20.00]	9 [20.00]	10 [20.00]	10 [20.00]	1 [20.00]	11 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	4 [33.33]	5 [33.33]	6 [33.33]
2 [12.50]	2 [12.50]	3 [12.50]	2 [12.50]	5 [12.50]	3 [12.50]	11 [12.50]	1 [12.50]	2 [12.50]	8 [20.00]	9 [20.00]	10 [20.00]
11 [12.50]	5 [12.50]	6 [12.50]							9 [20.00]	10 [20.00]	11 [20.00]
									1 [12.50]	2 [12.50]	3 [12.50]

to 220%												to 220%
to 210%												to 210%
to 200%												to 200%
to 190%												to 190%
to 180%												to 180%
to 170%												to 170%
to 160%												to 160%
to 150%												to 150%
to 140%												to 140%
to 130%												to 130%
to 120%												to 120%
to 110%												to 110%
to 100%												to 100%
to 90%												to 90%
to 80%												to 80%
to 70%												to 70%
to 60%	0-8-9 55.56 %	0-3-11 55.56 %	0-1-4 55.56 %									to 60%
to 50%												to 50%
to 40%												to 40%
to 30%												to 30%
to 20%												to 20%
to 10%				0-9-11 6.67 %	0-2-3 6.67 %	0-1-10 6.67 %	0-9-10 6.67 %	0-2-11 6.67 %	0-1-3 6.67 %	0-10-11 6.67 %	0-1-11 6.67 %	0-1-2 6.67 %	to 10%
0% / 2 equal roots													0% / 2 equal roots
0% / 3 equal roots													0% / 3 equal roots
0% / 4+ equal roots													0% / 4+ equal roots
Root clarity	TSC3-3A			TSC3-2B			TSC3-2A			TSC3-1			Root clarity

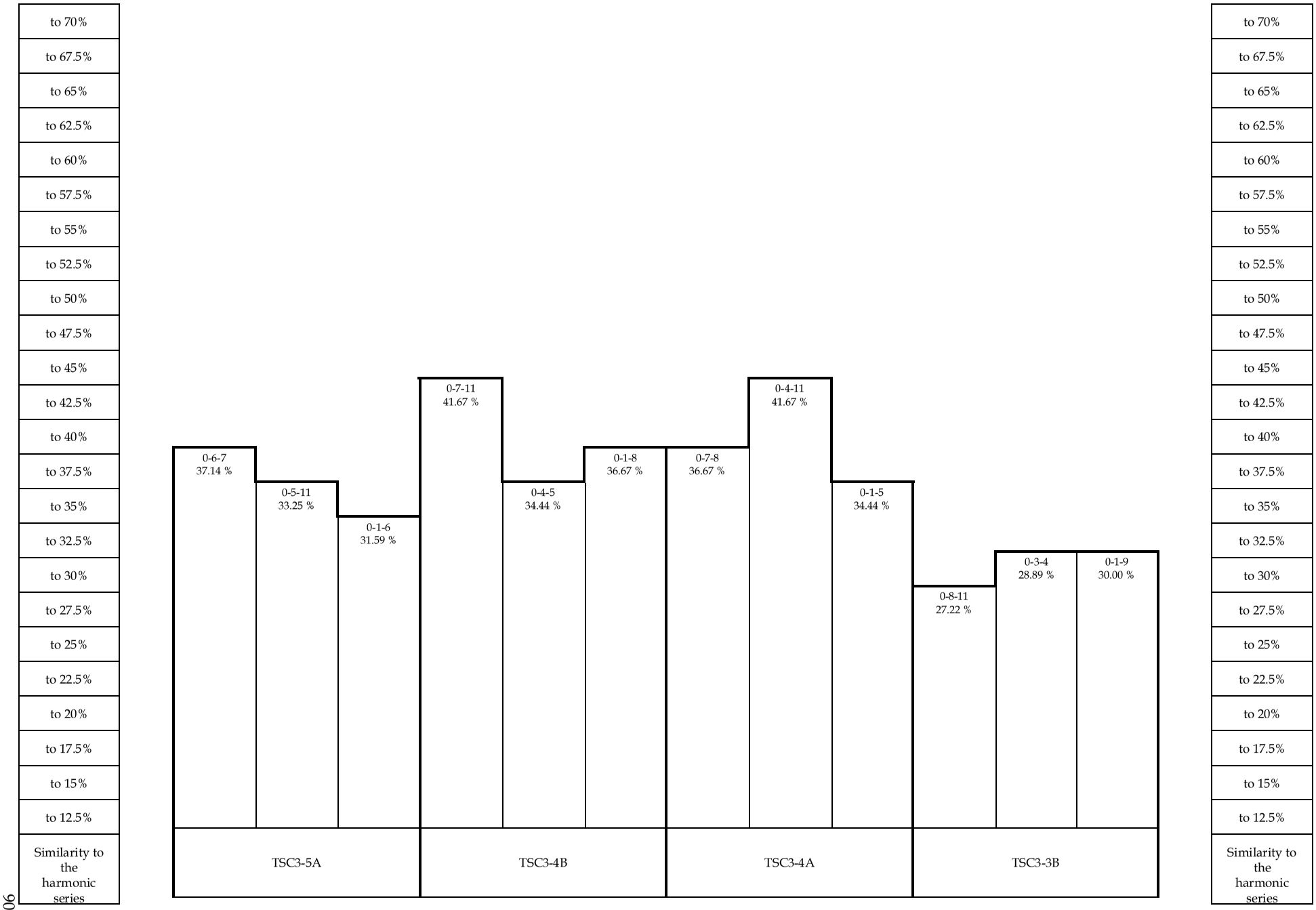
TSC3-3A			TSC3-2B			TSC3-2A			TSC3-1		
0-8-9	0-3-11	0-1-4	0-9-11	0-2-3	0-1-10	0-9-10	0-2-11	0-1-3	0-10-11	0-1-11	0-1-2
8 [155.56]	11 [155.56]	0 [155.56]	11 [140.00]	2 [140.00]	0 [140.00]	0 [140.00]	2 [140.00]	3 [140.00]	0 [140.00]	1 [140.00]	2 [140.00]
0 [100.00]	0 [100.00]	1 [100.00]	9 [131.25]	0 [131.25]	10 [131.25]	10 [131.25]	0 [131.25]	1 [131.25]	10 [131.25]	11 [131.25]	0 [131.25]
9 [100.00]	3 [100.00]	4 [100.00]	0 [100.00]	3 [100.00]	1 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]
5 [97.22]	8 [97.22]	9 [97.22]	5 [97.22]	8 [97.22]	6 [97.22]	5 [97.22]	7 [97.22]	8 [97.22]	8 [53.47]	9 [53.47]	10 [53.47]
2 [81.67]	5 [81.67]	6 [81.67]	2 [81.67]	5 [81.67]	3 [81.67]	2 [81.67]	4 [81.67]	5 [81.67]	3 [50.00]	4 [50.00]	5 [50.00]
1 [50.00]	4 [50.00]	5 [50.00]	7 [53.47]	10 [53.47]	8 [53.47]	8 [53.47]	10 [53.47]	11 [53.47]	4 [50.00]	5 [50.00]	6 [50.00]
10 [37.92]	1 [37.92]	2 [37.92]	4 [50.00]	7 [50.00]	5 [50.00]	3 [50.00]	5 [50.00]	6 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]
4 [33.33]	7 [33.33]	8 [33.33]	8 [33.33]	11 [33.33]	9 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	6 [33.33]	7 [33.33]	8 [33.33]
11 [20.00]	2 [20.00]	3 [20.00]	1 [20.00]	4 [20.00]	2 [20.00]	11 [20.00]	1 [20.00]	2 [20.00]	7 [33.33]	8 [33.33]	9 [33.33]
6 [12.50]	9 [12.50]	10 [12.50]	10 [12.50]	1 [12.50]	11 [12.50]	7 [12.50]	9 [12.50]	10 [12.50]	1 [20.00]	2 [20.00]	3 [20.00]
7 [12.50]	10 [12.50]	11 [12.50]							2 [20.00]	3 [20.00]	4 [20.00]
									9 [12.50]	10 [12.50]	11 [12.50]

Names

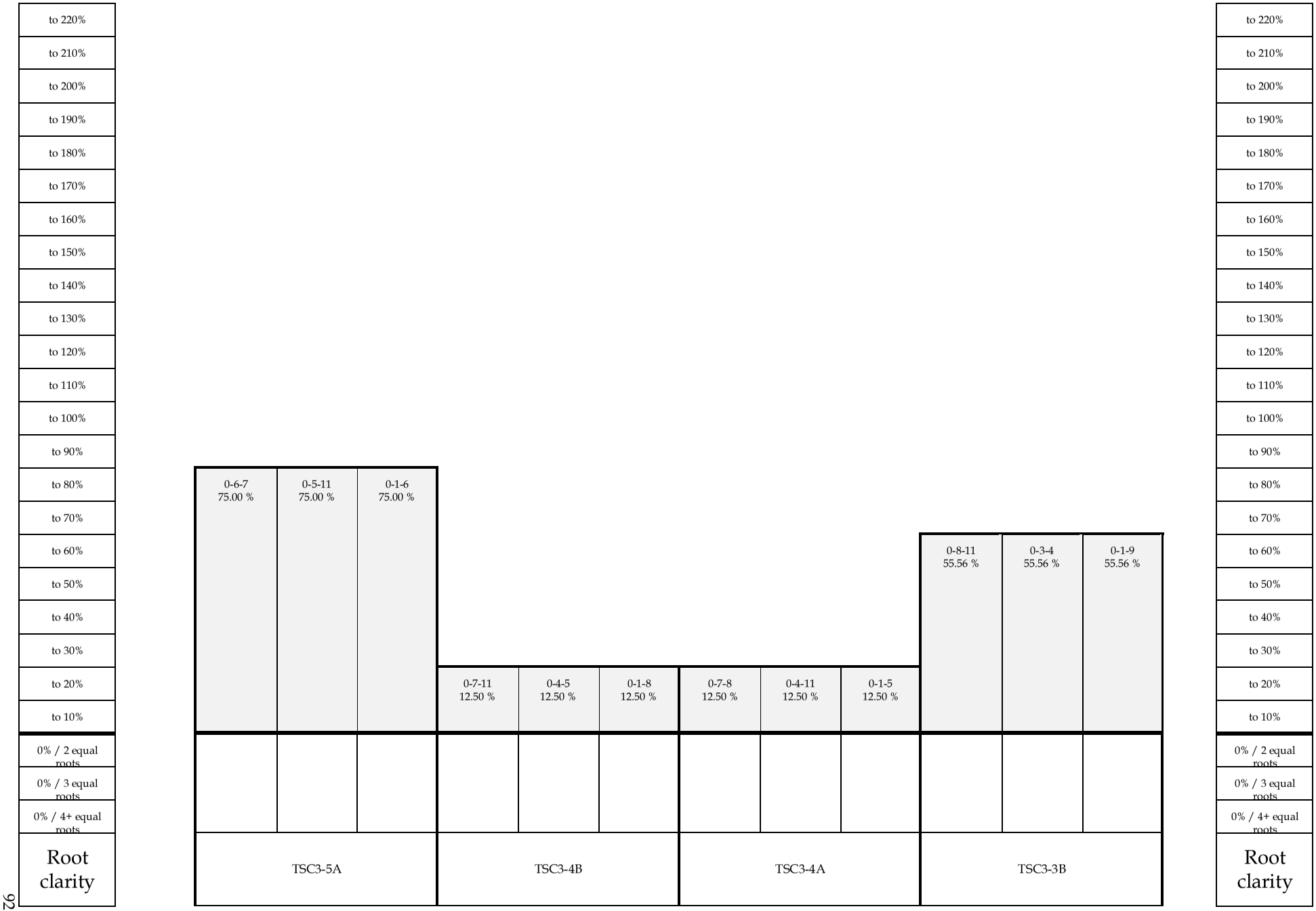
0-8-9	0-8-9 trichord	0-3-11	0-3-11 trichord	0-1-4	0-1-4 trichord	0-9-11	0-9-11 trichord	0-2-3	0-2-3 trichord	0-1-10	0-1-10 trichord
0-9-10	0-9-10 trichord	0-2-11	0-2-11 trichord	0-1-3	0-1-3 trichord	0-10-11	0-10-11 trichord	0-1-11	0-1-11 trichord	0-1-2	Chromatic cluster
TSC3-3A		TSC3-2B		TSC3-2A		TSC3-1					

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-6-7
37.14 %
0-5-11
33.25 %
0-1-6
31.59 %
0-7-11
41.67 %
0-4-5
34.44 %
0-1-8
36.67 %
0-7-8
36.67 %
0-4-11
41.67 %
0-1-5
34.44 %
0-8-11
27.22 %
0-3-4
28.89 %
0-1-9
30.00 %
TSC3-5A
TSC3-4B
TSC3-4A
TSC3-3B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC3-5A			TSC3-4B			TSC3-4A			TSC3-3B		
0-6-7	0-5-11	0-1-6	0-7-11	0-4-5	0-1-8	0-7-8	0-4-11	0-1-5	0-8-11	0-3-4	0-1-9
7 [150.00]	0 [150.00]	1 [150.00]	7 [150.00]	0 [150.00]	8 [150.00]	7 [150.00]	11 [150.00]	0 [150.00]	0 [133.33]	4 [133.33]	1 [133.33]
0 [100.00]	5 [100.00]	0 [100.00]	11 [133.33]	4 [133.33]	0 [133.33]	0 [133.33]	4 [133.33]	5 [133.33]	8 [100.00]	0 [100.00]	0 [100.00]
6 [100.00]	11 [100.00]	6 [100.00]	0 [100.00]	5 [100.00]	1 [100.00]	8 [100.00]	0 [100.00]	1 [100.00]	11 [100.00]	3 [100.00]	9 [100.00]
2 [62.50]	7 [62.50]	8 [62.50]	2 [62.50]	7 [62.50]	3 [62.50]	2 [62.50]	6 [62.50]	7 [62.50]	3 [83.33]	7 [83.33]	4 [83.33]
4 [53.33]	3 [53.33]	4 [53.33]	6 [50.00]	11 [50.00]	7 [50.00]	3 [50.00]	7 [50.00]	8 [50.00]	6 [70.00]	10 [70.00]	7 [70.00]
10 [53.33]	9 [53.33]	10 [53.33]	3 [33.33]	8 [33.33]	4 [33.33]	4 [33.33]	3 [33.33]	4 [33.33]	7 [50.00]	11 [50.00]	8 [50.00]
1 [50.00]	6 [50.00]	7 [50.00]	4 [33.33]	9 [33.33]	5 [33.33]	11 [33.33]	8 [33.33]	9 [33.33]	4 [33.33]	8 [33.33]	5 [33.33]
11 [33.33]	4 [33.33]	5 [33.33]	9 [32.50]	2 [32.50]	10 [32.50]	10 [32.50]	2 [32.50]	3 [32.50]	10 [32.50]	2 [32.50]	11 [32.50]
5 [20.00]	10 [20.00]	11 [20.00]	5 [20.00]	3 [20.00]	6 [20.00]	5 [20.00]	9 [20.00]	10 [20.00]	9 [20.00]	1 [20.00]	10 [20.00]
8 [12.50]	1 [12.50]	2 [12.50]	10 [20.00]	10 [20.00]	11 [20.00]	6 [20.00]	10 [20.00]	11 [20.00]	1 [12.50]	5 [12.50]	2 [12.50]
9 [12.50]	2 [12.50]	3 [12.50]	1 [12.50]	6 [12.50]	2 [12.50]	9 [12.50]	1 [12.50]	2 [12.50]	2 [12.50]	6 [12.50]	3 [12.50]


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-6-7 75.00 %
0-5-11 75.00 %
0-1-6 75.00 %
0-7-11 12.50 %
0-4-5 12.50 %
0-1-8 12.50 %
0-7-8 12.50 %
0-4-11 12.50 %
0-1-5 12.50 %
0-8-11 55.56 %
0-3-4 55.56 %
0-1-9 55.56 %
TSC3-5A
TSC3-4B
TSC3-4A
TSC3-3B
Root clarity

TSC3-5A				TSC3-4B				TSC3-4A				TSC3-3B			
0-6-7	0-5-11	0-1-6		0-7-11	0-4-5	0-1-8		0-7-8	0-4-11	0-1-5		0-8-11	0-3-4	0-1-9	
0 [175.00]	5 [175.00]	6 [175.00]		0 [175.00]	5 [175.00]	1 [175.00]		0 [175.00]	4 [175.00]	5 [175.00]		8 [155.56]	0 [155.56]	9 [155.56]	
6 [100.00]	0 [100.00]	0 [100.00]		7 [155.56]	0 [155.56]	8 [155.56]		8 [155.56]	0 [155.56]	1 [155.56]		0 [100.00]	3 [100.00]	0 [100.00]	
7 [100.00]	11 [100.00]	1 [100.00]		11 [100.00]	4 [100.00]	0 [100.00]		7 [100.00]	11 [100.00]	0 [100.00]		11 [100.00]	4 [100.00]	1 [100.00]	
5 [72.92]	10 [72.92]	11 [72.92]		5 [72.92]	10 [72.92]	6 [72.92]		5 [72.92]	9 [72.92]	10 [72.92]		4 [97.22]	8 [97.22]	5 [97.22]	
2 [62.22]	1 [62.22]	2 [62.22]		4 [50.00]	9 [50.00]	5 [50.00]		1 [50.00]	5 [50.00]	6 [50.00]		1 [81.67]	5 [81.67]	2 [81.67]	
8 [62.22]	7 [62.22]	8 [62.22]		9 [37.92]	2 [37.92]	10 [37.92]		10 [37.92]	2 [37.92]	3 [37.92]		5 [50.00]	9 [50.00]	6 [50.00]	
11 [50.00]	4 [50.00]	5 [50.00]		3 [33.33]	1 [33.33]	4 [33.33]		3 [33.33]	7 [33.33]	8 [33.33]		10 [37.92]	2 [37.92]	11 [37.92]	
3 [33.33]	8 [33.33]	9 [33.33]		8 [33.33]	8 [33.33]	9 [33.33]		4 [33.33]	8 [33.33]	9 [33.33]		7 [33.33]	11 [33.33]	8 [33.33]	
9 [20.00]	2 [20.00]	3 [20.00]		1 [20.00]	6 [20.00]	2 [20.00]		2 [20.00]	1 [20.00]	2 [20.00]		2 [20.00]	6 [20.00]	3 [20.00]	
4 [12.50]	3 [12.50]	4 [12.50]		2 [20.00]	7 [20.00]	3 [20.00]		9 [20.00]	6 [20.00]	7 [20.00]		6 [12.50]	1 [12.50]	7 [12.50]	
10 [12.50]	9 [12.50]	10 [12.50]		10 [12.50]	3 [12.50]	11 [12.50]		6 [12.50]	10 [12.50]	11 [12.50]		9 [12.50]	10 [12.50]	10 [12.50]	

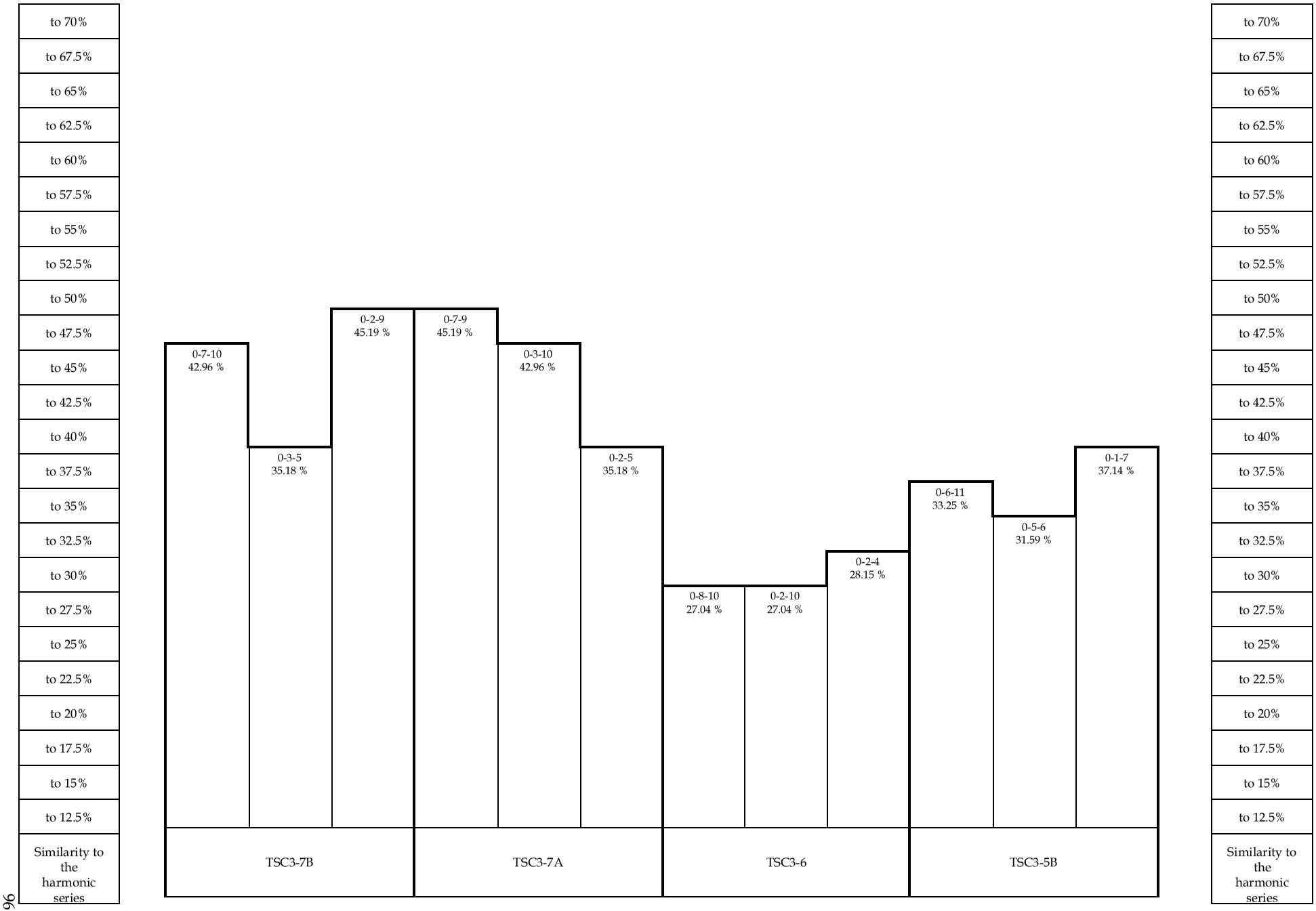
Root candidates and weights

Names

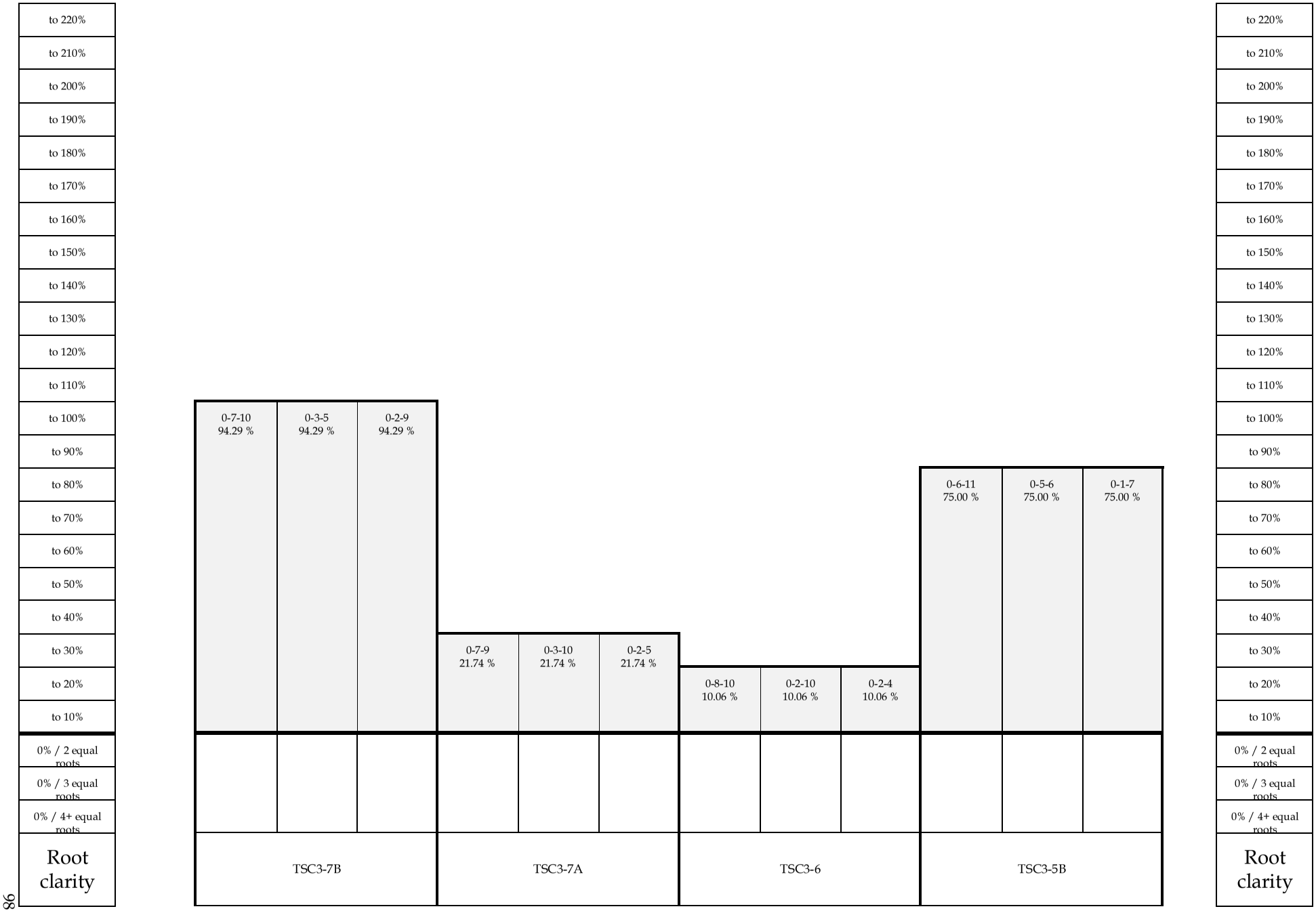
0-6-7 0-6-7 trichord	0-5-11 Fifth scale degree Balinese pentatonic triad	0-1-6 Webern trichord	0-7-11 0-7-11 trichord	0-4-5 0-4-5 trichord	0-1-8 0-1-8 trichord	0-7-8 0-7-8 trichord	0-4-11 0-4-11 trichord	0-1-5 0-1-5 trichord	0-8-11 0-8-11 trichord	0-3-4 0-3-4 trichord	0-1-9 0-1-9 trichord			
TSC3-5A			TSC3-4B			TSC3-4A			TSC3-3B					

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-7-10
42.96 %
0-3-5
35.18 %
0-2-9
45.19 %
0-7-9
45.19 %
0-3-10
42.96 %
0-2-5
35.18 %
0-8-10
27.04 %
0-2-10
27.04 %
0-2-4
28.15 %
0-6-11
33.25 %
0-5-6
31.59 %
0-1-7
37.14 %
TSC3-7B
TSC3-7A
TSC3-6
TSC3-5B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC3-7B				TSC3-7A				TSC3-6				TSC3-5B			
0-7-10	0-3-5	0-2-9		0-7-9	0-3-10	0-2-5		0-8-10	0-2-10	0-2-4		0-6-11	0-5-6	0-1-7	
7 [150.00]	0 [150.00]	9 [150.00]		7 [170.00]	10 [170.00]	0 [170.00]		0 [145.83]	2 [145.83]	4 [145.83]		6 [150.00]	0 [150.00]	7 [150.00]	
10 [120.00]	3 [120.00]	0 [120.00]		9 [112.50]	0 [112.50]	2 [112.50]		10 [132.50]	0 [132.50]	2 [132.50]		0 [100.00]	5 [100.00]	0 [100.00]	
0 [112.50]	5 [112.50]	2 [112.50]		0 [100.00]	3 [100.00]	5 [100.00]		8 [120.00]	10 [120.00]	0 [120.00]		11 [100.00]	6 [100.00]	1 [100.00]	
2 [95.83]	7 [95.83]	4 [95.83]		4 [83.33]	7 [83.33]	9 [83.33]		3 [50.00]	5 [50.00]	7 [50.00]		1 [62.50]	7 [62.50]	2 [62.50]	
5 [70.00]	10 [70.00]	7 [70.00]		2 [62.50]	5 [62.50]	7 [62.50]		5 [50.00]	7 [50.00]	9 [50.00]		4 [53.33]	4 [53.33]	5 [53.33]	
4 [33.33]	4 [33.33]	1 [33.33]		11 [45.83]	2 [45.83]	4 [45.83]		7 [50.00]	9 [50.00]	11 [50.00]		10 [53.33]	10 [53.33]	11 [53.33]	
11 [33.33]	9 [33.33]	6 [33.33]		1 [33.33]	4 [33.33]	6 [33.33]		2 [45.83]	4 [45.83]	6 [45.83]		7 [50.00]	1 [50.00]	8 [50.00]	
8 [20.00]	1 [20.00]	10 [20.00]		5 [20.00]	1 [20.00]	3 [20.00]		4 [33.33]	6 [33.33]	8 [33.33]		3 [33.33]	9 [33.33]	4 [33.33]	
9 [12.50]	2 [12.50]	11 [12.50]		10 [20.00]	8 [20.00]	10 [20.00]]		6 [20.00]	8 [20.00]	10 [20.00]		9 [20.00]	3 [20.00]	10 [20.00]	
												2 [12.50]	2 [12.50]	3 [12.50]	
												8 [12.50]	8 [12.50]	9 [12.50]	


to 220%
to 210%
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to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-7-10 94.29 %
0-3-5 94.29 %
0-2-9 94.29 %
0-7-9 21.74 %
0-3-10 21.74 %
0-2-5 21.74 %
0-8-10 10.06 %
0-2-10 10.06 %
0-2-4 10.06 %
0-6-11 75.00 %
0-5-6 75.00 %
0-1-7 75.00 %
TSC3-7B
TSC3-7A
TSC3-6
TSC3-5B
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

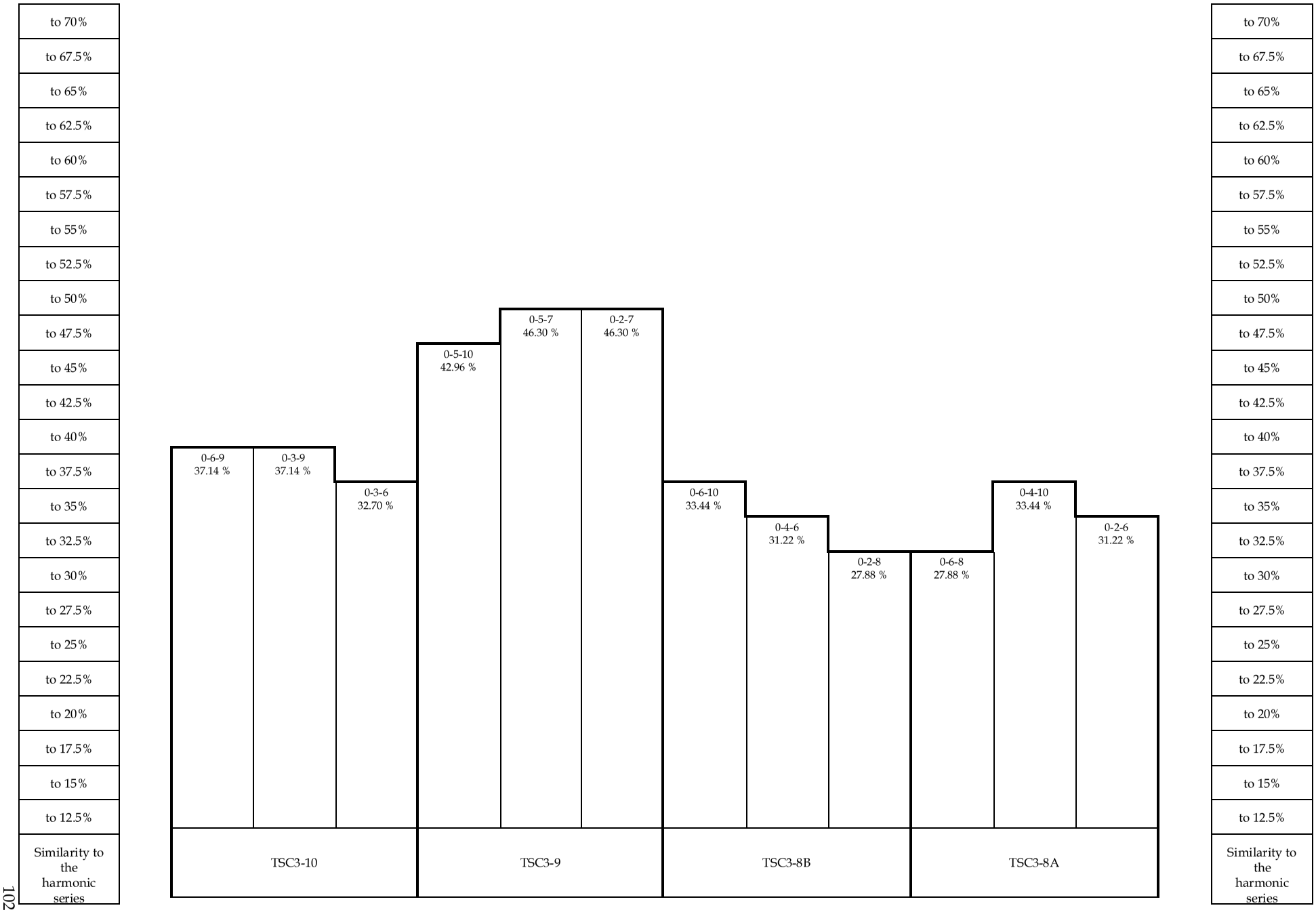
TSC3-7B				TSC3-7A				TSC3-6				TSC3-5B			
0-7-10	0-3-5	0-2-9		0-7-9	0-3-10	0-2-5		0-8-10	0-2-10	0-2-4		0-6-11	0-5-6	0-1-7	
0 [255.00]	5 [255.00]	2 [255.00]		0 [175.00]	3 [175.00]	5 [175.00]		8 [218.75]	10 [218.75]	0 [218.75]		11 [175.00]	5 [175.00]	0 [175.00]	
10 [131.25]	3 [131.25]	0 [131.25]		5 [143.75]	8 [143.75]	10 [143.75]		10 [198.75]	0 [198.75]	2 [198.75]		0 [100.00]	0 [100.00]	1 [100.00]	
7 [100.00]	0 [100.00]	9 [100.00]		9 [140.00]	0 [140.00]	2 [140.00]		0 [140.00]	2 [140.00]	4 [140.00]		6 [100.00]	6 [100.00]	7 [100.00]	
3 [97.22]	8 [97.22]	5 [97.22]		7 [131.25]	10 [131.25]	0 [131.25]		6 [53.47]	8 [53.47]	10 [53.47]		4 [72.92]	10 [72.92]	5 [72.92]	
5 [72.92]	10 [72.92]	7 [72.92]		2 [81.67]	5 [81.67]	7 [81.67]		1 [50.00]	3 [50.00]	5 [50.00]		2 [62.22]	2 [62.22]	3 [62.22]	
8 [53.47]	1 [53.47]	10 [53.47]		3 [33.33]	6 [33.33]	1 [33.33]		3 [50.00]	5 [50.00]	7 [50.00]		8 [62.22]	8 [62.22]	9 [62.22]	
6 [33.33]	11 [33.33]	8 [33.33]		8 [33.33]	11 [33.33]	8 [33.33]		5 [50.00]	7 [50.00]	9 [50.00]		5 [50.00]	11 [50.00]	6 [50.00]	
2 [20.00]	2 [20.00]	4 [20.00]		11 [20.00]	2 [20.00]	4 [20.00]		4 [33.33]	6 [33.33]	8 [33.33]		7 [33.33]	1 [33.33]	8 [33.33]	
9 [20.00]	7 [20.00]	11 [20.00]		10 [12.50]	1 [12.50]	3 [12.50]		2 [20.00]	4 [20.00]	6 [20.00]		1 [20.00]	7 [20.00]	2 [20.00]	
												9 [12.50]	3 [12.50]	10 [12.50]	
												10 [12.50]	4 [12.50]	11 [12.50]	

Names

0-1-7 0-1-7 trichord														TSC3-5B
0-5-6 0-5-6 trichord														
0-6-11 Second scale degree Balinese pentatonic triad														
0-2-4 0-2-4 trichord														TSC3-6
0-2-10 0-2-10 trichord														
0-8-10 0-8-10 trichord														
0-2-5 0-2-5 trichord														TSC3-7A
0-3-10 Ute Tritonic scale (Native North American) m3m7														
0-7-9 0-7-9 trichord														
0-2-9 0-2-9 trichord														TSC3-7B
0-3-5 0-3-5 trichord														
0-7-10 0-7-10 trichord P5m7														

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
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to 35%
to 32.5%
to 30%
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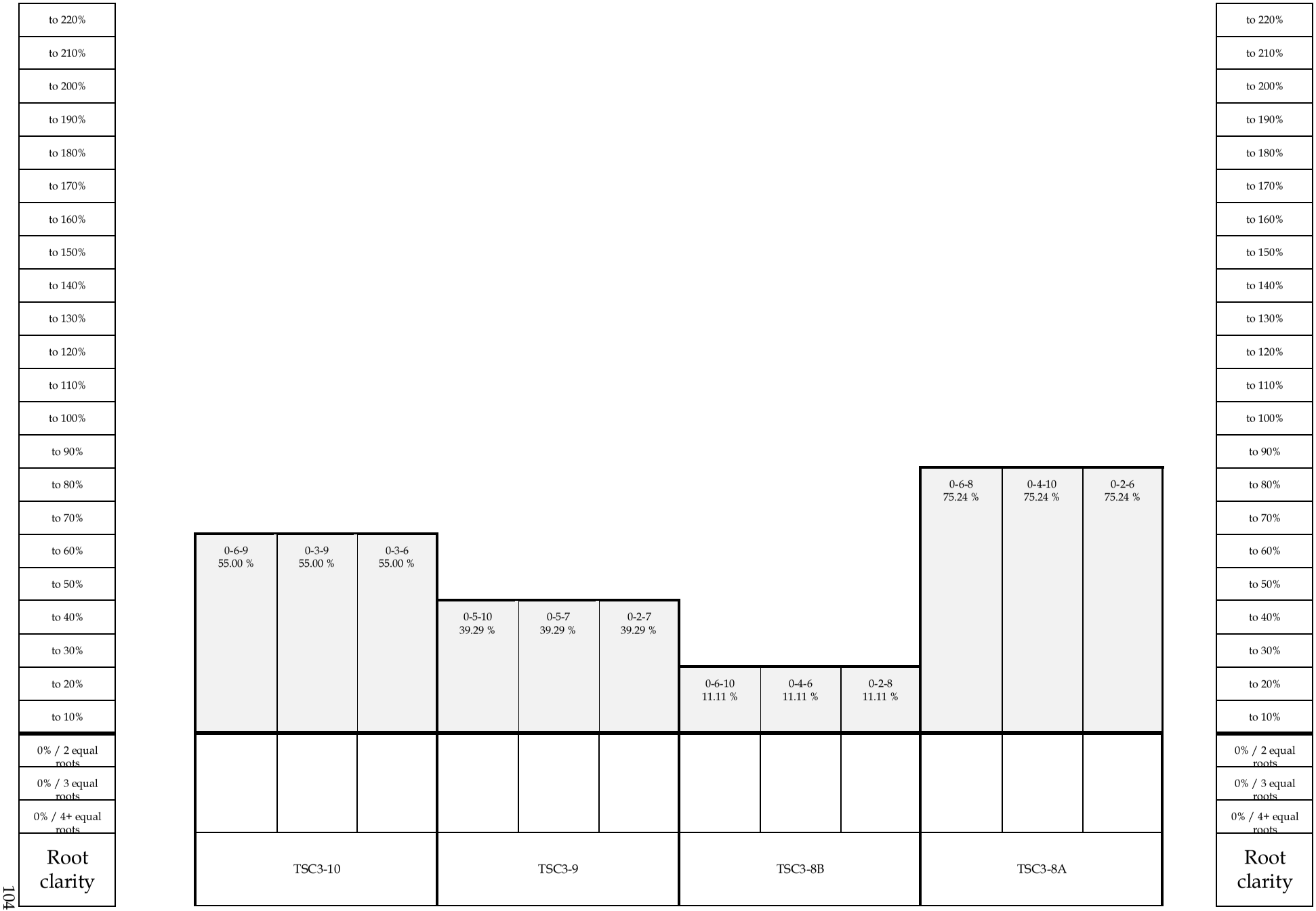
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-6-9 37.14 %
0-3-9 37.14 %
0-3-6 32.70 %
0-5-10 42.96 %
0-5-7 46.30 %
0-2-7 46.30 %
0-6-10 33.44 %
0-4-6 31.22 %
0-2-8 27.88 %
0-6-8 27.88 %
0-4-10 33.44 %
0-2-6 31.22 %
TSC3-10
TSC3-9
TSC3-8B
TSC3-8A

TSC3-10				TSC3-9			TSC3-8B			TSC3-8A		
0-6-9	0-3-9	0-3-6	0-5-10	0-5-7	0-2-7	0-6-10	0-4-6	0-2-8	0-6-8	0-4-10	0-2-6
4 [103.33]	7 [103.33]	10 [103.33]	0 [162.50]	7 [162.50]	2 [162.50]	10 [153.33]	4 [153.33]	0 [153.33]	0 [133.33]	4 [133.33]	6 [133.33]
0 [100.00]	0 [100.00]	0 [100.00]	5 [150.00]	0 [150.00]	7 [150.00]	0 [112.50]	6 [112.50]	2 [112.50]	6 [120.00]	10 [120.00]	0 [120.00]
6 [100.00]	3 [100.00]	3 [100.00]	10 [120.00]	5 [120.00]	0 [120.00]	6 [100.00]	0 [100.00]	8 [100.00]	8 [112.50]	0 [112.50]	2 [112.50]
9 [100.00]	9 [100.00]	6 [100.00]	7 [62.50]	2 [62.50]	9 [62.50]	4 [53.33]	10 [53.33]	6 [53.33]	10 [65.83]	2 [65.83]	4 [65.83]
1 [83.33]	4 [83.33]	7 [83.33]	2 [45.83]	9 [45.83]	4 [45.83]	1 [50.00]	1 [50.00]	3 [50.00]	4 [53.33]	8 [53.33]	10 [53.33]
7 [70.00]	10 [70.00]	1 [70.00]	4 [33.33]	4 [33.33]	6 [33.33]	5 [50.00]	7 [50.00]	7 [50.00]	1 [50.00]	5 [50.00]	1 [50.00]
10 [53.33]	1 [53.33]	4 [53.33]	9 [33.33]	11 [33.33]	11 [33.33]	7 [50.00]	11 [50.00]	9 [50.00]	3 [50.00]	7 [50.00]	7 [50.00]
2 [12.50]	2 [12.50]	2 [12.50]	3 [20.00]	3 [20.00]	5 [20.00]	2 [45.83]	8 [45.83]	4 [45.83]	7 [50.00]	11 [50.00]	9 [50.00]
8 [12.50]	5 [12.50]	5 [12.50]	8 [20.00]	10 [20.00]	10 [20.00]	8 [32.50]	2 [32.50]	10 [32.50]	2 [12.50]	6 [12.50]	8 [12.50]
11 [12.50]	11 [12.50]	8 [12.50]									


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
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to 120%
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to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-6-9 55.00 %
0-3-9 55.00 %
0-3-6 55.00 %
0-5-10 39.29 %
0-5-7 39.29 %
0-2-7 39.29 %
0-6-10 11.11 %
0-4-6 11.11 %
0-2-8 11.11 %
0-6-8 75.24 %
0-4-10 75.24 %
0-2-6 75.24 %
TSC3-10
TSC3-9
TSC3-8B
TSC3-8A
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

TSC3-10				TSC3-9			TSC3-8B			TSC3-8A		
0-6-9	0-3-9	0-3-6	0-5-10	0-5-7	0-2-7	0-6-10	0-4-6	0-2-8	0-6-8	0-4-10	0-2-6
2 [155.00]	5 [155.00]	8 [155.00]	10 [243.75]	5 [243.75]	0 [243.75]	6 [155.56]	0 [155.56]	8 [155.56]	8 [230.00]	0 [230.00]	2 [230.00]
0 [100.00]	0 [100.00]	0 [100.00]	5 [175.00]	0 [175.00]	7 [175.00]	0 [140.00]	6 [140.00]	2 [140.00]	6 [131.25]	10 [131.25]	0 [131.25]
6 [100.00]	3 [100.00]	3 [100.00]	0 [140.00]	7 [140.00]	2 [140.00]	10 [131.25]	4 [131.25]	0 [131.25]	0 [100.00]	4 [100.00]	6 [100.00]
9 [100.00]	9 [100.00]	6 [100.00]	3 [72.92]	10 [72.92]	5 [72.92]	8 [98.75]	2 [98.75]	10 [98.75]	2 [62.22]	6 [62.22]	8 [62.22]
5 [97.22]	8 [97.22]	11 [97.22]	8 [53.47]	3 [53.47]	10 [53.47]	2 [62.22]	8 [62.22]	4 [62.22]	4 [53.47]	8 [53.47]	10 [53.47]
11 [81.67]	2 [81.67]	5 [81.67]	1 [33.33]	1 [33.33]	3 [33.33]	3 [50.00]	5 [50.00]	1 [50.00]	1 [50.00]	3 [50.00]	5 [50.00]
8 [62.22]	11 [62.22]	2 [62.22]	6 [33.33]	8 [33.33]	8 [33.33]	5 [50.00]	9 [50.00]	5 [50.00]	5 [50.00]	5 [50.00]	7 [50.00]
4 [12.50]	1 [12.50]	1 [12.50]	2 [20.00]	2 [20.00]	4 [20.00]	11 [50.00]	11 [50.00]	7 [50.00]	11 [50.00]	9 [50.00]	11 [50.00]
7 [12.50]	7 [12.50]	4 [12.50]	7 [20.00]	9 [20.00]	9 [20.00]	4 [12.50]	10 [12.50]	6 [12.50]	10 [37.92]	2 [37.92]	4 [37.92]
10 [12.50]	10 [12.50]	10 [12.50]									

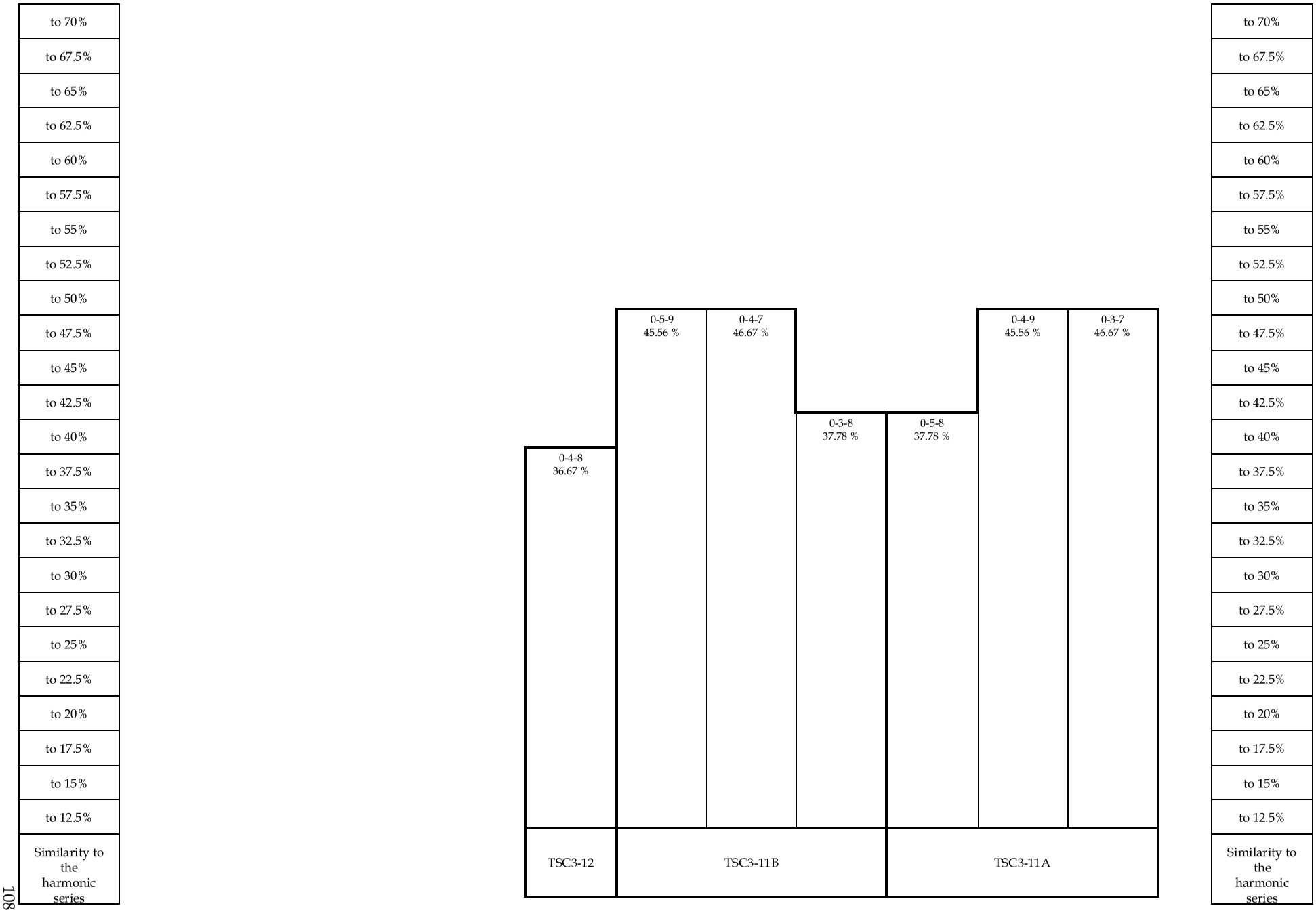
Root candidates and weights

Names

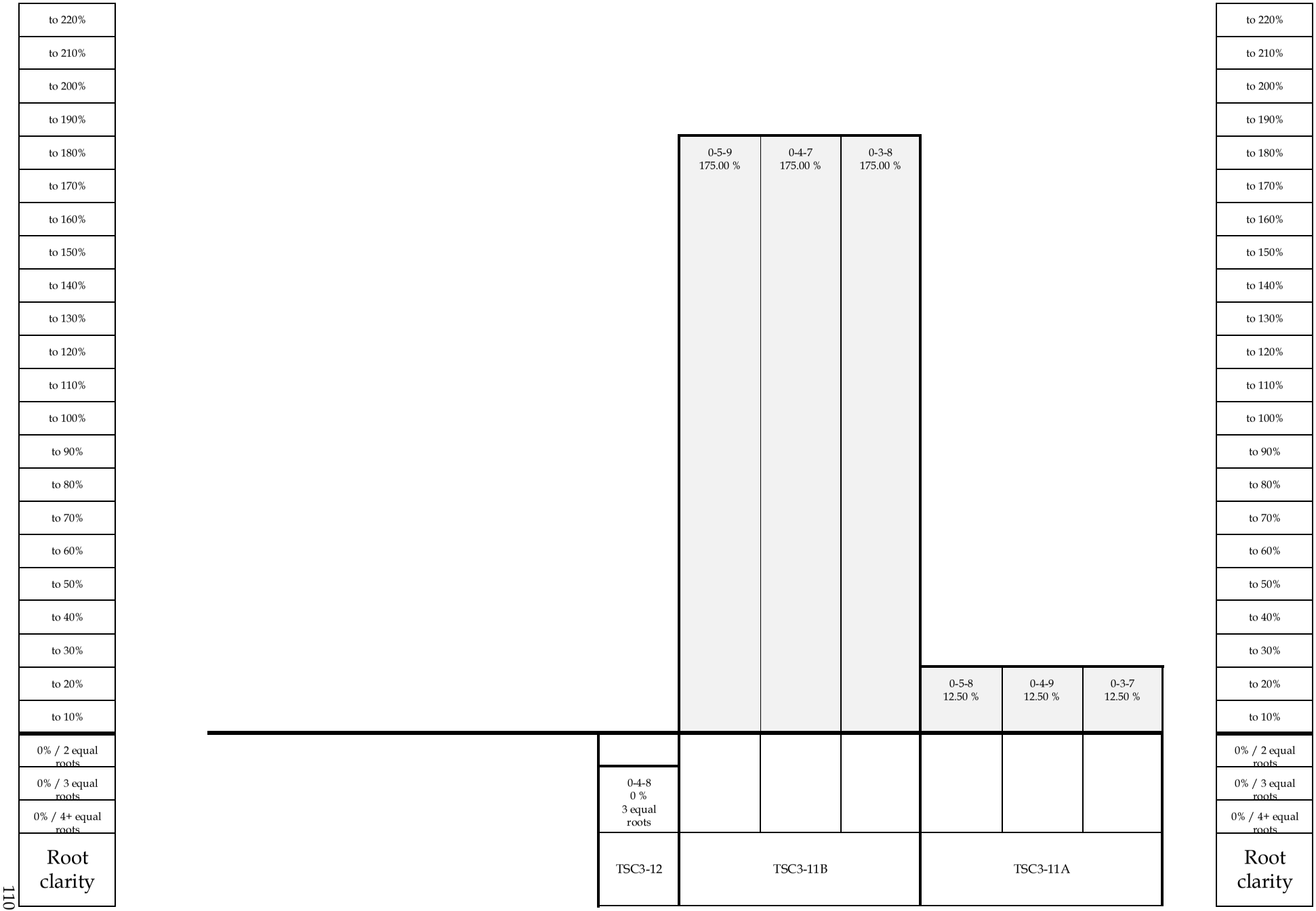
0-6-9	0-3-9	0-3-6	0-5-10	0-5-7	0-2-7	0-6-10	0-4-6	0-2-8	0-6-8	0-4-10	0-2-6
Second inversion diminished triad	First inversion diminished triad	Diminished triad	Perfect quartal trichord	Suspended triad	Add 9 omit 3 trichord	Second inversion major flat 5	Major flat 5	First inversion major flat 5	First inversion Italian sixth	Italian sixth	Second inversion Italian sixth
			Sansagari scale (Japan)	sus	Ilerici chord	d5m7	Major flat 5th			M3m7	
			First scale degree pentatonic minor triad	Three-pitch harmonic similarity scale							
			Second scale degree Chinese-Mongolian pentatonic triad	Warao Tritonic scale (Native South America)							
			Third scale degree pentatonic minor triad								
			Third scale degree Chinese-Mongolian pentatonic triad								
			Third scale degree Balinese pentatonic triad								
			Fourth scale degree pentatonic minor triad								
			Fifth scale degree Chinese-Mongolian pentatonic triad								
TSC3-10			TSC3-9			TSC3-8B			TSC3-8A		

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-4-8
36.67 %
0-5-9
45.56 %
0-4-7
46.67 %
0-3-8
37.78 %
0-5-8
37.78 %
0-4-9
45.56 %
0-3-7
46.67 %
TSC3-12
TSC3-11B
TSC3-11A
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC3-12	TSC3-11B				TSC3-11A		
0-4-8	0-5-9	0-4-7	0-3-8	0-5-8	0-4-9	0-3-7
0 [133.33] 4 [133.33] 8 [133.33] 3 [50.00] 7 [50.00] 11 [50.00] 2 [32.50] 6 [32.50] 10 [32.50]	0 [150.00] 9 [133.33] 5 [100.00] 4 [83.33] 7 [82.50] 1 [33.33] 3 [20.00] 10 [20.00] 2 [12.50] 11 [12.50]	7 [150.00] 4 [133.33] 0 [100.00] 11 [83.33] 2 [82.50] 8 [33.33] 5 [20.00] 10 [20.00] 6 [12.50] 9 [12.50]	3 [150.00] 0 [133.33] 8 [100.00] 7 [83.33] 10 [82.50] 4 [33.33] 1 [20.00] 6 [20.00] 2 [12.50] 5 [12.50]	0 [183.33] 5 [100.00] 8 [100.00] 3 [70.00] 7 [62.50] 4 [33.33] 9 [33.33] 10 [32.50] 6 [20.00] 2 [12.50]	4 [183.33] 0 [100.00] 9 [100.00] 7 [70.00] 11 [62.50] 1 [33.33] 8 [33.33] 2 [32.50] 10 [20.00] 6 [12.50]	7 [183.33] 0 [100.00] 3 [100.00] 10 [70.00] 2 [62.50] 4 [33.33] 11 [33.33] 5 [32.50] 1 [20.00] 9 [12.50]


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-4-8
0 %
3 equal roots
TSC3-12
0-5-9
175.00 %
0-4-7
175.00 %
0-3-8
175.00 %
TSC3-11B
0-5-8
12.50 %
0-4-9
12.50 %
0-3-7
12.50 %
TSC3-11A
Root clarity

TSC3-12	TSC3-11B					TSC3-11A			
0-4-8	0-5-9	0-4-7	0-3-8	0-5-8	0-4-9	0-3-7			
0 [155.56]	5 [275.00]	0 [275.00]	8 [275.00]	5 [175.00]	9 [175.00]	0 [175.00]			
4 [155.56]	0 [100.00]	4 [100.00]	0 [100.00]	8 [155.56]	0 [155.56]	3 [155.56]			
8 [155.56]	9 [100.00]	7 [100.00]	3 [100.00]	10 [123.75]	2 [123.75]	5 [123.75]			
1 [50.00]	2 [81.67]	9 [81.67]	5 [81.67]	0 [100.00]	4 [100.00]	7 [100.00]			
5 [50.00]	10 [72.92]	5 [72.92]	1 [72.92]	1 [97.22]	5 [97.22]	8 [97.22]			
9 [50.00]	7 [37.92]	2 [37.92]	10 [37.92]	4 [33.33]	8 [33.33]	11 [33.33]			
2 [37.92]	1 [33.33]	3 [33.33]	4 [33.33]	2 [20.00]	6 [20.00]	2 [20.00]			
6 [37.92]	8 [33.33]	8 [33.33]	11 [33.33]	7 [20.00]	11 [20.00]	9 [20.00]			
10 [37.92]	11 [20.00]	6 [20.00]	2 [20.00]	3 [12.50]	7 [12.50]	1 [12.50]			
	3 [12.50]	10 [12.50]	6 [12.50]	6 [12.50]	10 [12.50]	10 [12.50]			

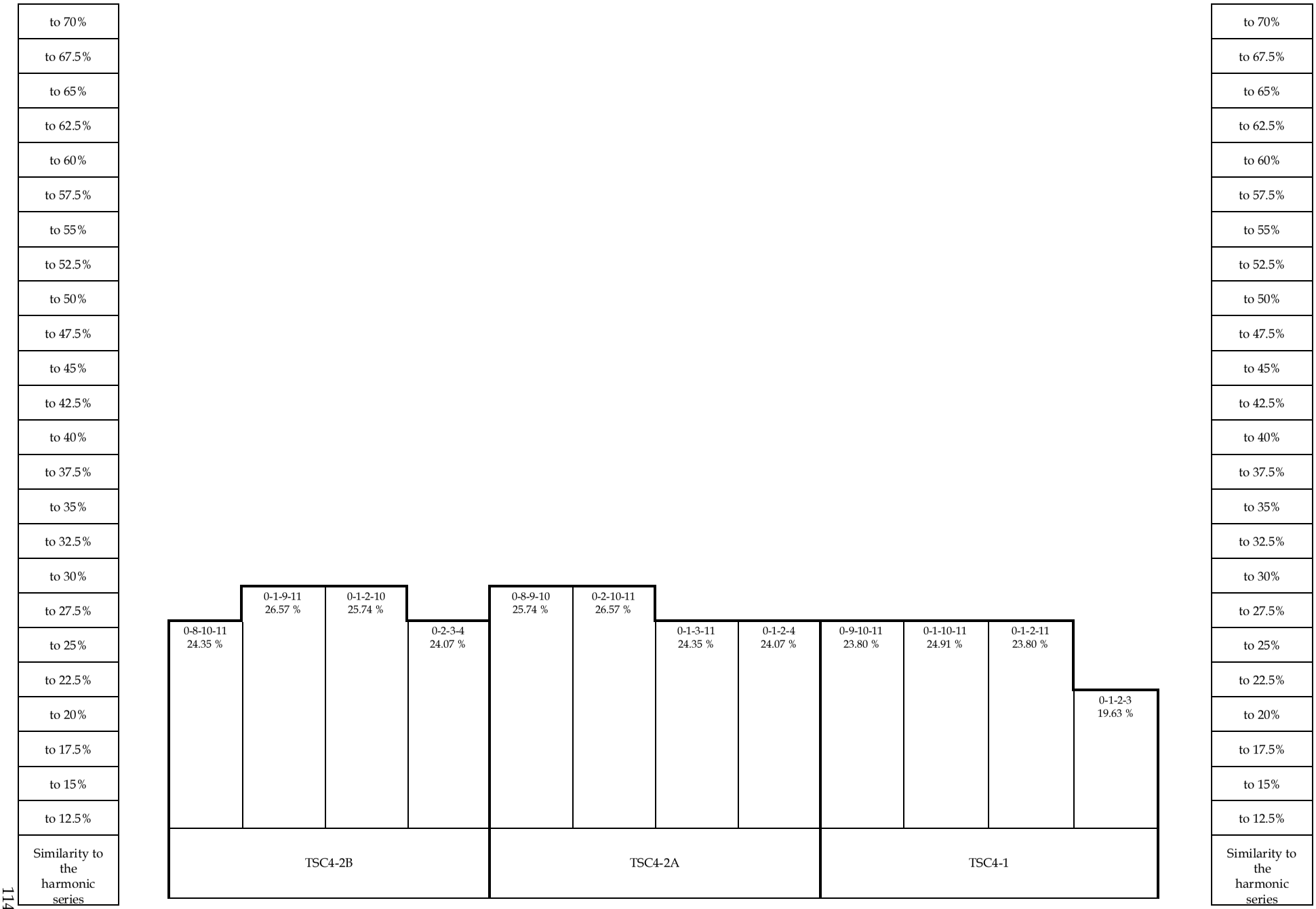
Root candidates and weights

Names

0-3-7	0-4-9	0-5-8	0-3-8	0-4-7	0-5-9	0-4-8
Minor triad	First inversion minor triad	Second inversion minor triad	First inversion major triad	Major triad	Second inversion major triad	Augmented triad
	First scale degree Chinese-Mongolian pentatonic triad	Fourth scale degree Balinese pentatonic triad	6 3 or 6 First scale degree Balinese pentatonic triad	5 3 Neapolitan sixth Peruvian Tritonic 1 scale	6 4 Tonic 6/4 chord Fourth scale degree Chinese-Mongolian pentatonic triad Fifth scale degree pentatonic minor triad	
	Second scale degree pentatonic minor triad					
TSC3-11A			TSC3-11B			TSC3-12

Names

Four-pitch chords
(seventh chords and tetrachords)


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-8-10-11
24.35 %
0-1-9-11
26.57 %
0-1-2-10
25.74 %
0-2-3-4
24.07 %
0-8-9-10
25.74 %
0-2-10-11
26.57 %
0-1-3-11
24.35 %
0-1-2-4
24.07 %
0-9-10-11
23.80 %
0-1-10-11
24.91 %
0-1-2-11
23.80 %
0-1-2-3
19.63 %
TSC4-2B
TSC4-2A
TSC4-1
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-2B				TSC4-2A				TSC4-1			
0-8-10-11	0-1-9-11	0-1-2-10	0-2-3-4	0-8-9-10	0-2-10-11	0-1-3-11	0-1-2-4	0-9-10-11	0-1-10-11	0-1-2-11	0-1-2-3
0 [145.83]	1 [145.83]	2 [145.83]	4 [145.83]	0 [145.83]	2 [145.83]	3 [145.83]	4 [145.83]	9 [120.00]	10 [120.00]	0 [120.00]	0 [120.00]
10 [132.50]	11 [132.50]	0 [132.50]	2 [132.50]	10 [132.50]	0 [132.50]	1 [132.50]	2 [132.50]	10 [120.00]	11 [120.00]	11 [120.00]	1 [120.00]
8 [120.00]	9 [120.00]	10 [120.00]	0 [120.00]	8 [120.00]	10 [120.00]	11 [120.00]	0 [120.00]	0 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]
11 [100.00]	0 [100.00]	1 [100.00]	3 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]	11 [112.50]	1 [112.50]	2 [112.50]	3 [112.50]
3 [83.33]	4 [83.33]	5 [83.33]	7 [83.33]	4 [83.33]	6 [83.33]	7 [83.33]	8 [83.33]	4 [83.33]	5 [83.33]	6 [83.33]	7 [83.33]
6 [70.00]	7 [70.00]	8 [70.00]	10 [70.00]	7 [70.00]	9 [70.00]	10 [70.00]	11 [70.00]	7 [70.00]	8 [70.00]	9 [70.00]	10 [70.00]
5 [50.00]	6 [50.00]	7 [50.00]	9 [50.00]	3 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]	8 [50.00]
7 [50.00]	8 [50.00]	9 [50.00]	11 [50.00]	5 [50.00]	7 [50.00]	8 [50.00]	9 [50.00]	6 [50.00]	7 [50.00]	8 [50.00]	9 [50.00]
2 [45.83]	3 [45.83]	4 [45.83]	6 [45.83]	2 [45.83]	4 [45.83]	5 [45.83]	6 [45.83]	1 [45.83]	2 [45.83]	3 [45.83]	4 [45.83]
4 [33.33]	5 [33.33]	6 [33.33]	8 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	5 [33.33]	2 [45.83]	3 [45.83]	4 [45.83]	5 [45.83]
9 [20.00]	10 [20.00]	11 [20.00]	1 [20.00]	6 [20.00]	8 [20.00]	9 [20.00]	10 [20.00]	3 [33.33]	4 [33.33]	5 [33.33]	6 [33.33]
1 [12.50]	2 [12.50]	3 [12.50]	5 [12.50]	11 [12.50]	1 [12.50]	2 [12.50]	3 [12.50]	8 [20.00]	9 [20.00]	10 [20.00]	11 [20.00]

to 220%											to 220%
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to 60%											to 60%
to 50%											to 50%
to 40%											to 40%
to 30%											to 30%
to 20%	0-8-10-11 10.06 %	0-1-9-11 10.06 %	0-1-2-10 10.06 %	0-2-3-4 10.06 %	0-8-9-10 10.06 %	0-2-10-11 10.06 %	0-1-3-11 10.06 %	0-1-2-4 10.06 %			to 20%
to 10%											to 10%
0% / 2 equal roots									0-9-10-11 0 % 2 equal roots	0-1-10-11 0 % 2 equal roots	0-1-2-11 0 % 2 equal roots	0-1-2-3 0 % 2 equal roots	0% / 2 equal roots
0% / 3 equal roots													0% / 3 equal roots
0% / 4+ equal roots													0% / 4+ equal roots
Root clarity	TSC4-2B				TSC4-2A				TSC4-1				Root clarity

TSC4-2B				TSC4-2A				TSC4-1			
0-8-10-11	0-1-9-11	0-1-2-10	0-2-3-4	0-8-9-10	0-2-10-11	0-1-3-11	0-1-2-4	0-9-10-11	0-1-10-11	0-1-2-11	0-1-2-3
8 [218.75] 10 [198.75] 0 [140.00] 11 [100.00] 4 [97.22] 1 [81.67] 6 [53.47] 3 [50.00] 5 [50.00] 7 [33.33] 2 [20.00] 9 [12.50]	9 [218.75] 11 [198.75] 1 [140.00] 0 [100.00] 5 [97.22] 2 [81.67] 7 [53.47] 4 [50.00] 6 [50.00] 8 [33.33] 3 [20.00] 10 [12.50]	10 [218.75] 0 [198.75] 2 [140.00] 1 [100.00] 6 [97.22] 3 [81.67] 8 [53.47] 5 [50.00] 7 [50.00] 9 [33.33] 4 [20.00] 11 [12.50]	0 [218.75] 2 [198.75] 4 [140.00] 3 [100.00] 8 [97.22] 5 [81.67] 10 [53.47] 7 [50.00] 9 [50.00] 11 [33.33] 6 [20.00] 1 [12.50]	8 [218.75] 10 [198.75] 0 [140.00] 9 [100.00] 5 [97.22] 2 [81.67] 6 [53.47] 1 [50.00] 3 [50.00] 4 [33.33] 11 [20.00] 7 [12.50]	10 [218.75] 0 [198.75] 2 [140.00] 11 [100.00] 7 [97.22] 4 [81.67] 8 [53.47] 3 [50.00] 5 [50.00] 6 [33.33] 1 [20.00] 9 [12.50]	11 [218.75] 1 [198.75] 3 [140.00] 0 [100.00] 8 [97.22] 5 [81.67] 9 [53.47] 4 [50.00] 6 [50.00] 7 [33.33] 2 [20.00] 10 [12.50]	0 [218.75] 2 [198.75] 4 [140.00] 1 [100.00] 9 [97.22] 6 [81.67] 10 [53.47] 5 [50.00] 7 [50.00] 8 [33.33] 3 [20.00] 11 [12.50]	0 [140.00] 11 [140.00] 9 [131.25] 10 [131.25] 5 [97.22] 2 [81.67] 7 [53.47] 8 [53.47] 3 [50.00] 4 [50.00] 6 [33.33] 1 [20.00]	0 [140.00] 1 [140.00] 10 [131.25] 11 [131.25] 6 [97.22] 3 [81.67] 8 [53.47] 9 [53.47] 4 [50.00] 5 [50.00] 7 [33.33] 2 [20.00]	1 [140.00] 2 [140.00] 0 [131.25] 11 [131.25] 7 [97.22] 4 [81.67] 9 [53.47] 10 [53.47] 5 [50.00] 6 [50.00] 8 [33.33] 3 [20.00]	2 [140.00] 3 [140.00] 0 [131.25] 1 [131.25] 8 [97.22] 5 [81.67] 10 [53.47] 11 [53.47] 6 [50.00] 7 [50.00] 9 [33.33] 4 [20.00]

Names

0-8-10-11 0-8-10-11 tetrachord	0-1-9-11 0-1-9-11 tetrachord	0-1-2-10 0-1-2-10 tetrachord	0-2-3-4 0-2-3-4 tetrachord	0-8-9-10 0-8-9-10 tetrachord	0-2-10-11 0-2-10-11 tetrachord	0-1-3-11 0-1-3-11 tetrachord	0-1-2-4 0-1-2-4 tetrachord	0-9-10-11 0-9-10-11 tetrachord	0-1-10-11 0-1-10-11 tetrachord	0-1-2-11 0-1-2-11 tetrachord	0-1-2-3 0-1-2-3 tetrachord 0-1-2-3 cluster
TSC4-2B				TSC4-2A				TSC4-1			

Names

Appendix A continues on the next page.

to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-3-4-5
28.70 %
0-7-10-11
33.43 %
0-1-8-11
31.76 %
0-1-2-9
32.04 %
0-7-8-9
32.04 %
0-3-10-11
31.76 %
0-1-4-11
33.43 %
0-1-2-5
28.70 %
0-8-9-11
27.31 %
0-1-9-10
29.82 %
0-2-3-11
27.31 %
0-1-3-4
25.92 %
TSC4-4B
TSC4-4A
TSC4-3
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
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to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-4B				TSC4-4A				TSC4-3			
0-3-4-5	0-7-10-11	0-1-8-11	0-1-2-9	0-7-8-9	0-3-10-11	0-1-4-11	0-1-2-5	0-8-9-11	0-1-9-10	0-2-3-11	0-1-3-4
0 [150.00]	7 [150.00]	8 [150.00]	9 [150.00]	7 [170.00]	10 [170.00]	11 [170.00]	0 [170.00]	0 [133.33]	1 [133.33]	3 [133.33]	4 [133.33]
4 [133.33]	11 [133.33]	0 [133.33]	1 [133.33]	0 [133.33]	3 [133.33]	4 [133.33]	5 [133.33]	9 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]
3 [120.00]	10 [120.00]	11 [120.00]	0 [120.00]	9 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]	11 [112.50]	0 [112.50]	2 [112.50]	3 [112.50]
5 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]	8 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]	8 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]
7 [95.83]	2 [95.83]	3 [95.83]	4 [95.83]	4 [83.33]	7 [83.33]	8 [83.33]	9 [83.33]	3 [83.33]	4 [83.33]	6 [83.33]	7 [83.33]
10 [70.00]	5 [70.00]	6 [70.00]	7 [70.00]	2 [62.50]	5 [62.50]	6 [62.50]	7 [62.50]	4 [83.33]	5 [83.33]	7 [83.33]	8 [83.33]
11 [50.00]	6 [50.00]	7 [50.00]	8 [50.00]	3 [50.00]	6 [50.00]	7 [50.00]	8 [50.00]	6 [70.00]	7 [70.00]	9 [70.00]	10 [70.00]
8 [33.33]	3 [33.33]	4 [33.33]	5 [33.33]	11 [45.83]	2 [45.83]	3 [45.83]	4 [45.83]	7 [70.00]	8 [70.00]	10 [70.00]	11 [70.00]
9 [33.33]	4 [33.33]	5 [33.33]	6 [33.33]	1 [33.33]	4 [33.33]	5 [33.33]	6 [33.33]	1 [45.83]	2 [45.83]	4 [45.83]	5 [45.83]
2 [32.50]	9 [32.50]	10 [32.50]	11 [32.50]	10 [32.50]	1 [32.50]	2 [32.50]	3 [32.50]	10 [32.50]	11 [32.50]	1 [32.50]	2 [32.50]
1 [20.00]	8 [20.00]	9 [20.00]	10 [20.00]	5 [20.00]	8 [20.00]	9 [20.00]	10 [20.00]	2 [12.50]	3 [12.50]	5 [12.50]	6 [12.50]
6 [12.50]	1 [12.50]	2 [12.50]	3 [12.50]	6 [20.00]	9 [20.00]	10 [20.00]	11 [20.00]				

to 220%											
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to 80%											
to 70%	0-3-4-5 63.93 %	0-7-10-11 63.93 %	0-1-8-11 63.93 %	0-1-2-9 63.93 %							
to 60%											
to 50%											
to 40%											
to 30%											
to 20%					0-7-8-9 12.50 %	0-3-10-11 12.50 %	0-1-4-11 12.50 %	0-1-2-5 12.50 %	0-8-9-11 11.11 %	0-1-9-10 11.11 %	0-2-3-11 11.11 %	0-1-3-4 11.11 %
to 10%												
0% / 2 equal roots												
0% / 3 equal roots												
0% / 4+ equal roots												
Root clarity	TSC4-4B				TSC4-4A				TSC4-3			

Root clarity

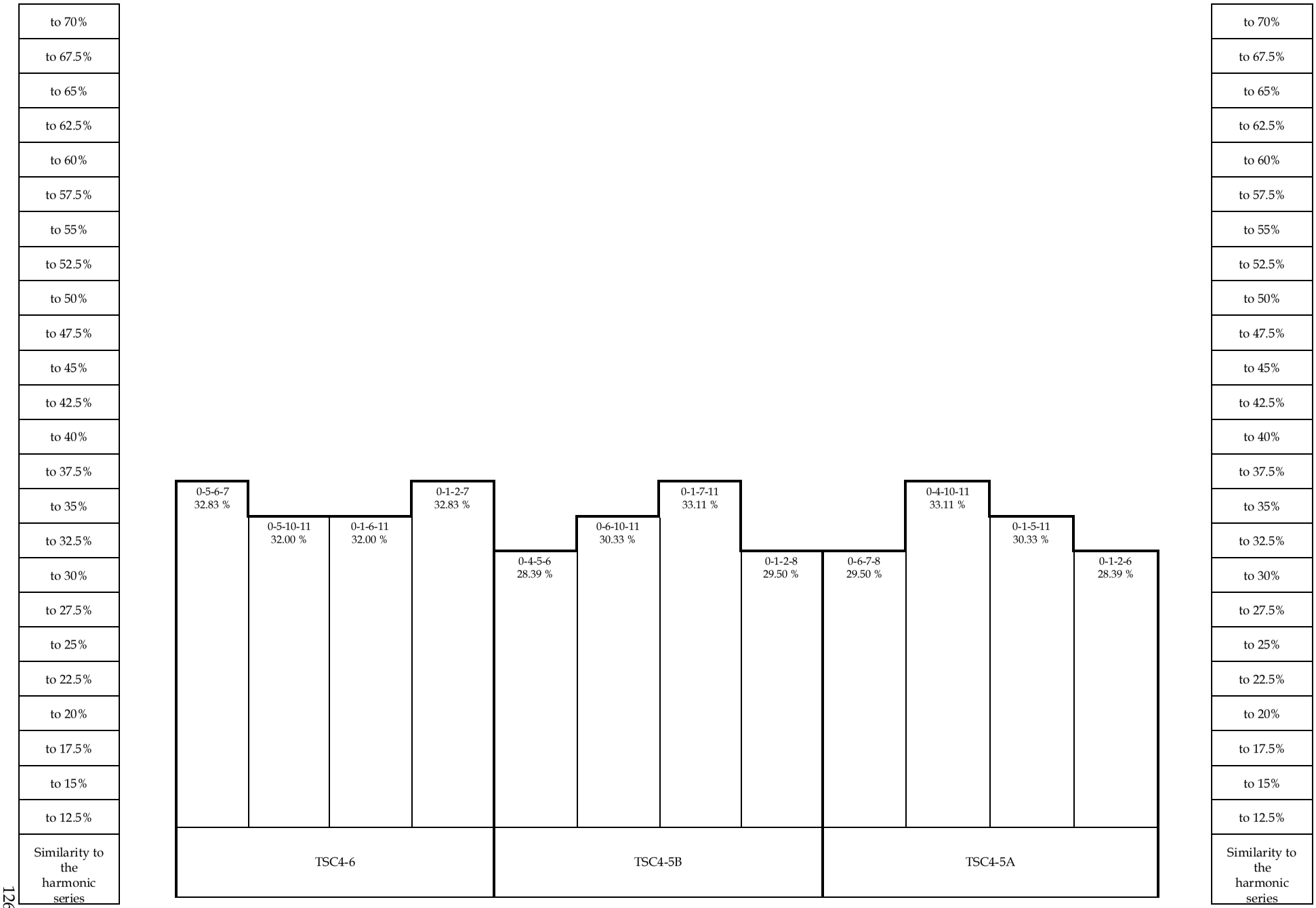
TSC4-4B				TSC4-4A				TSC4-3			
0-3-4-5	0-7-10-11	0-1-8-11	0-1-2-9	0-7-8-9	0-3-10-11	0-1-4-11	0-1-2-5	0-8-9-11	0-1-9-10	0-2-3-11	0-1-3-4
5 [255.00] 0 [155.56] 3 [131.25] 4 [100.00] 8 [97.22] 10 [72.92] 1 [53.47] 9 [50.00] 2 [37.92] 11 [33.33] 6 [20.00] 7 [20.00]	0 [255.00] 7 [155.56] 10 [131.25] 11 [100.00] 3 [97.22] 5 [72.92] 8 [53.47] 4 [50.00] 9 [37.92] 6 [33.33] 1 [20.00] 2 [20.00]	1 [255.00] 8 [155.56] 11 [131.25] 0 [100.00] 4 [97.22] 6 [72.92] 9 [53.47] 5 [50.00] 10 [37.92] 7 [33.33] 2 [20.00] 3 [20.00]	2 [255.00] 9 [155.56] 0 [131.25] 1 [100.00] 5 [97.22] 7 [72.92] 10 [53.47] 6 [50.00] 11 [37.92] 8 [33.33] 3 [20.00] 4 [20.00]	0 [175.00] 8 [155.56] 5 [143.75] 9 [140.00] 7 [131.25] 2 [81.67] 1 [50.00] 10 [37.92] 3 [33.33] 4 [33.33] 11 [20.00] 6 [12.50]	3 [175.00] 11 [155.56] 8 [143.75] 0 [140.00] 10 [131.25] 5 [81.67] 4 [50.00] 1 [37.92] 6 [33.33] 7 [33.33] 2 [20.00] 9 [12.50]	4 [175.00] 0 [155.56] 9 [143.75] 1 [140.00] 11 [131.25] 6 [81.67] 5 [50.00] 2 [37.92] 7 [33.33] 8 [33.33] 3 [20.00] 10 [12.50]	5 [175.00] 1 [155.56] 10 [143.75] 2 [140.00] 0 [131.25] 7 [81.67] 6 [50.00] 3 [37.92] 8 [33.33] 9 [33.33] 4 [20.00] 11 [12.50]	8 [155.56] 11 [140.00] 9 [131.25] 0 [100.00] 4 [97.22] 5 [97.22] 1 [81.67] 2 [81.67] 7 [53.47] 10 [37.92] 6 [12.50]	9 [155.56] 0 [140.00] 10 [131.25] 1 [100.00] 5 [97.22] 6 [97.22] 2 [81.67] 3 [81.67] 8 [53.47] 11 [37.92] 7 [12.50]	11 [155.56] 2 [140.00] 0 [131.25] 3 [100.00] 7 [97.22] 8 [97.22] 4 [81.67] 5 [81.67] 10 [53.47] 1 [37.92] 9 [12.50]	0 [155.56] 3 [140.00] 1 [131.25] 4 [100.00] 8 [97.22] 9 [97.22] 5 [81.67] 6 [81.67] 11 [53.47] 2 [37.92] 10 [12.50]

Names

TSC4-3	TSC4-4A	TSC4-4B
0-1-3-4 0-1-3-4 tetrachord	0-7-8-9 0-7-8-9 tetrachord	0-3-4-5 0-3-4-5 tetrachord
0-2-3-11 0-2-3-11 tetrachord	0-3-10-11 0-3-10-11 tetrachord	0-7-10-11 0-7-10-11 tetrachord
0-1-9-10 0-1-9-10 tetrachord	0-1-4-11 0-1-4-11 tetrachord	0-1-8-11 0-1-8-11 tetrachord
0-8-9-11 0-8-9-11 tetrachord	0-1-2-5 0-1-2-5 tetrachord	0-1-2-9 0-1-2-9 tetrachord

Names

Appendix A continues on the next page.


to 70%
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to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-6-7
32.83 %
0-5-10-11
32.00 %
0-1-6-11
32.00 %
0-1-2-7
32.83 %
0-4-5-6
28.39 %
0-6-10-11
30.33 %
0-1-7-11
33.11 %
0-1-2-8
29.50 %
0-6-7-8
29.50 %
0-4-10-11
33.11 %
0-1-5-11
30.33 %
0-1-2-6
28.39 %
TSC4-6
TSC4-5B
TSC4-5A
to 70%
to 67.5%
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to 62.5%
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to 57.5%
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to 35%
to 32.5%
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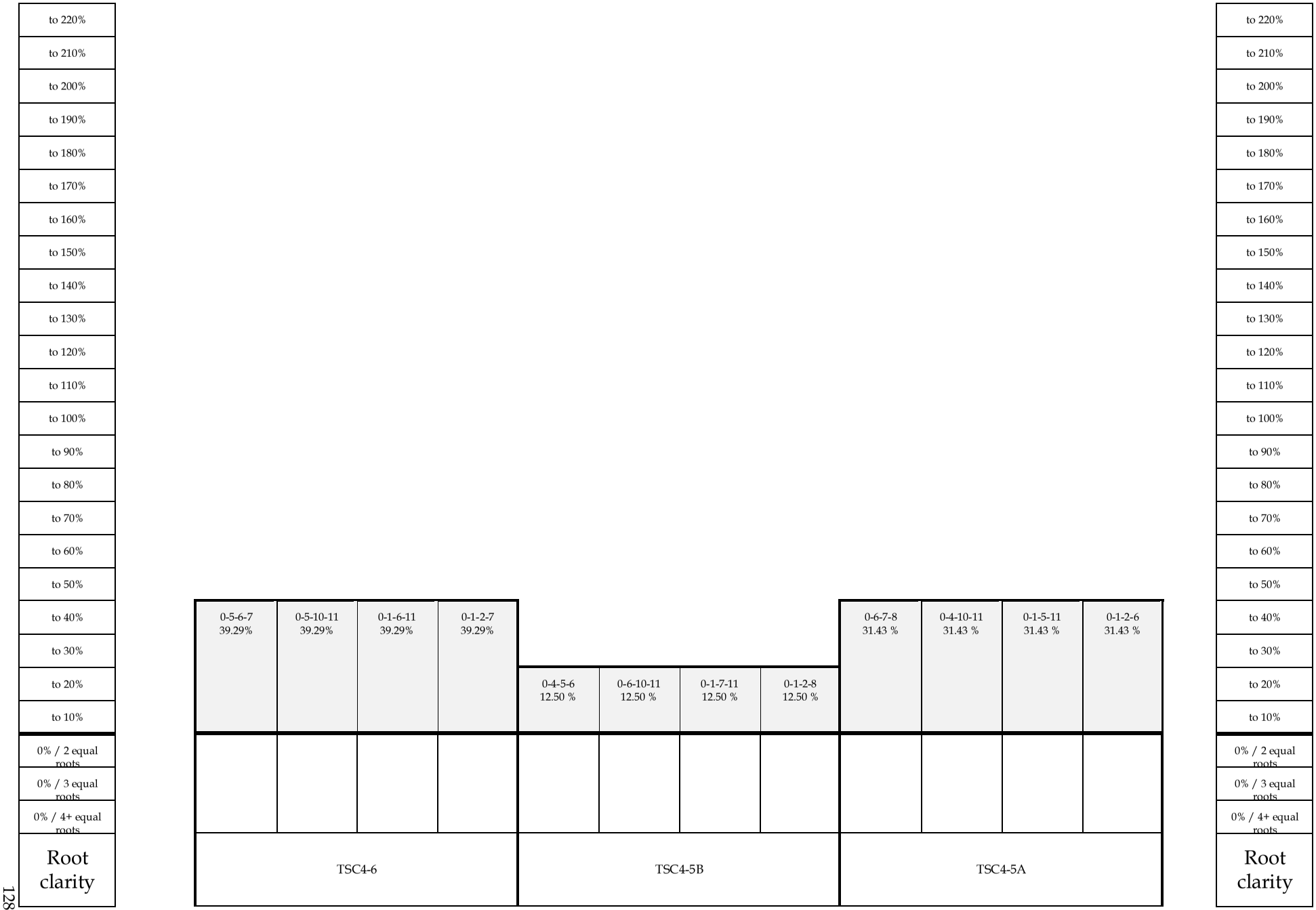
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-6				TSC4-5B				TSC4-5A			
0-5-6-7	0-5-10-11	0-1-6-11	0-1-2-7	0-4-5-6	0-6-10-11	0-1-7-11	0-1-2-8	0-6-7-8	0-4-10-11	0-1-5-11	0-1-2-6
7 [162.50]	0 [162.50]	1 [162.50]	2 [162.50]	4 [153.33]	10 [153.33]	11 [153.33]	0 [153.33]	7 [150.00]	11 [150.00]	0 [150.00]	1 [150.00]
0 [150.00]	5 [150.00]	6 [150.00]	7 [150.00]	0 [150.00]	6 [150.00]	7 [150.00]	8 [150.00]	0 [133.33]	4 [133.33]	5 [133.33]	6 [133.33]
5 [120.00]	10 [120.00]	11 [120.00]	0 [120.00]	6 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]	6 [120.00]	10 [120.00]	11 [120.00]	0 [120.00]
6 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]	5 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]	8 [112.50]	0 [112.50]	1 [112.50]	2 [112.50]
2 [62.50]	7 [62.50]	8 [62.50]	9 [62.50]	7 [62.50]	1 [62.50]	2 [62.50]	3 [62.50]	10 [65.83]	2 [65.83]	3 [65.83]	4 [65.83]
4 [53.33]	3 [53.33]	4 [53.33]	5 [53.33]	10 [53.33]	4 [53.33]	5 [53.33]	6 [53.33]	2 [62.50]	6 [62.50]	7 [62.50]	8 [62.50]
10 [53.33]	9 [53.33]	10 [53.33]	11 [53.33]	1 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]	4 [53.33]	8 [53.33]	9 [53.33]	10 [53.33]
1 [50.00]	6 [50.00]	7 [50.00]	8 [50.00]	11 [50.00]	7 [50.00]	8 [50.00]	9 [50.00]	1 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]
9 [45.83]	2 [50.00]	3 [50.00]	4 [45.83]	8 [45.83]	2 [45.83]	3 [45.83]	4 [45.83]	3 [50.00]	7 [50.00]	8 [50.00]	9 [50.00]
11 [33.33]	4 [45.83]	5 [45.83]	6 [33.33]	9 [33.33]	3 [33.33]	4 [33.33]	5 [33.33]	11 [33.33]	3 [33.33]	4 [33.33]	5 [33.33]
3 [20.00]	8 [33.33]	9 [33.33]	10 [20.00]	2 [32.50]	8 [32.50]	9 [32.50]	10 [32.50]	5 [20.00]	9 [20.00]	10 [20.00]	11 [20.00]
8 [12.50]	1 [20.00]	2 [20.00]	3 [12.50]	3 [20.00]	9 [20.00]	10 [20.00]	11 [20.00]	9 [12.50]	1 [12.50]	2 [12.50]	3 [12.50]
	1 [12.50]	2 [12.50]									


to 220%
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to 100%
to 90%
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to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-6-7 39.29%
0-5-10-11 39.29%
0-1-6-11 39.29%
0-1-2-7 39.29%
0-4-5-6 12.50 %
0-6-10-11 12.50 %
0-1-7-11 12.50 %
0-1-2-8 12.50 %
0-6-7-8 31.43 %
0-4-10-11 31.43 %
0-1-5-11 31.43 %
0-1-2-6 31.43 %
TSC4-6
TSC4-5B
TSC4-5A
Root clarity

TSC4-6				TSC4-5B				TSC4-5A			
0-5-6-7	0-5-10-11	0-1-6-11	0-1-2-7	0-4-5-6	0-6-10-11	0-1-7-11	0-1-2-8	0-6-7-8	0-4-10-11	0-1-5-11	0-1-2-6
5 [243.75]	10 [243.75]	11 [243.75]	0 [243.75]	5 [175.00]	11 [175.00]	0 [175.00]	1 [175.00]	8 [230.00]	0 [230.00]	1 [230.00]	2 [230.00]
0 [175.00]	5 [175.00]	6 [175.00]	7 [175.00]	0 [155.56]	6 [155.56]	7 [155.56]	8 [155.56]	0 [175.00]	4 [175.00]	5 [175.00]	6 [175.00]
7 [140.00]	0 [140.00]	1 [140.00]	2 [140.00]	6 [140.00]	0 [140.00]	1 [140.00]	2 [140.00]	6 [131.25]	10 [131.25]	11 [131.25]	0 [131.25]
6 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]	4 [131.25]	10 [131.25]	11 [131.25]	0 [131.25]	7 [100.00]	11 [100.00]	0 [100.00]	1 [100.00]
10 [72.92]	3 [72.92]	4 [72.92]	5 [72.92]	2 [98.75]	8 [98.75]	9 [98.75]	10 [98.75]	5 [72.92]	9 [72.92]	10 [72.92]	11 [72.92]
2 [62.22]	1 [62.22]	2 [62.22]	3 [62.22]	10 [72.92]	4 [72.92]	5 [72.92]	6 [72.92]	2 [62.22]	6 [62.22]	7 [62.22]	8 [62.22]
8 [62.22]	7 [62.22]	8 [62.22]	9 [62.22]	8 [62.22]	2 [62.22]	3 [62.22]	4 [62.22]	4 [53.47]	8 [53.47]	9 [53.47]	10 [53.47]
3 [53.47]	8 [53.47]	9 [53.47]	10 [53.47]	9 [50.00]	3 [50.00]	4 [50.00]	5 [50.00]	1 [50.00]	3 [50.00]	4 [50.00]	5 [50.00]
11 [50.00]	4 [50.00]	5 [50.00]	6 [50.00]	11 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]	11 [50.00]	5 [50.00]	6 [50.00]	7 [50.00]
1 [33.33]	6 [33.33]	7 [33.33]	8 [33.33]	1 [33.33]	7 [33.33]	8 [33.33]	9 [33.33]	10 [37.92]	2 [37.92]	3 [37.92]	4 [37.92]
9 [20.00]	2 [20.00]	3 [20.00]	4 [20.00]	7 [20.00]	1 [20.00]	2 [20.00]	3 [20.00]	3 [33.33]	7 [33.33]	8 [33.33]	9 [33.33]
4 [12.50]	9 [12.50]	10 [12.50]	11 [12.50]	3 [12.50]	9 [12.50]	10 [12.50]	11 [12.50]	9 [20.00]	1 [20.00]	2 [20.00]	3 [20.00]

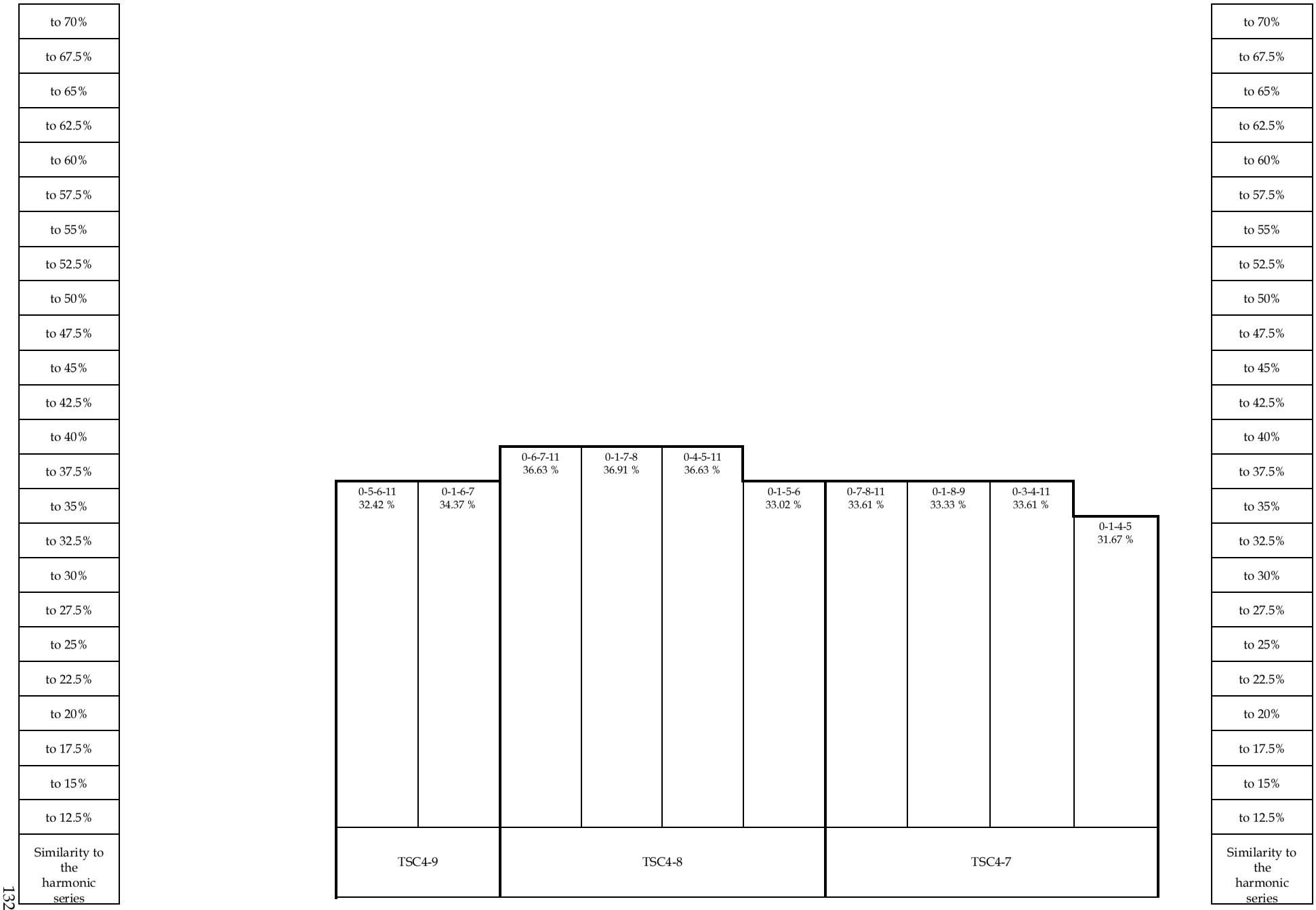
Root candidates and weights

Names

0-5-6-7 0-5-6-7 tetrachord	0-5-10-11 0-5-10-11 tetrachord	0-1-6-11 0-1-6-11 tetrachord	0-1-2-7 Dream chord	0-4-5-6 0-4-5-6 tetrachord	0-6-10-11 0-6-10-11 tetrachord	0-1-7-11 0-1-7-11 tetrachord	0-1-2-8 0-1-2-8 tetrachord	0-6-7-8 0-6-7-8 tetrachord	0-4-10-11 0-4-10-11 tetrachord	0-1-5-11 0-1-5-11 tetrachord	0-1-2-6 0-1-2-6 tetrachord
											TSC4-5A
											TSC4-5B
											TSC4-6

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-6-11 32.42 %
0-1-6-7 34.37 %
0-6-7-11 36.63 %
0-1-7-8 36.91 %
0-4-5-11 36.63 %
0-1-5-6 33.02 %
0-7-8-11 33.61 %
0-1-8-9 33.33 %
0-3-4-11 33.61 %
0-1-4-5 31.67 %
TSC4-9
TSC4-8
TSC4-7
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-9		TSC4-8				TSC4-7			
0-5-6-11	0-1-6-7	0-6-7-11	0-1-7-8	0-4-5-11	0-1-5-6	0-7-8-11	0-1-8-9	0-3-4-11	0-1-4-5
0 [150.00]	1 [150.00]	6 [150.00]	7 [150.00]	0 [150.00]	0 [150.00]	7 [150.00]	8 [150.00]	11 [150.00]	0 [150.00]
6 [150.00]	7 [150.00]	7 [150.00]	8 [150.00]	11 [150.00]	1 [150.00]	0 [133.33]	0 [133.33]	3 [133.33]	4 [133.33]
5 [100.00]	0 [100.00]	11 [133.33]	0 [133.33]	4 [133.33]	5 [133.33]	11 [133.33]	1 [133.33]	4 [133.33]	5 [133.33]
11 [100.00]	6 [100.00]	0 [100.00]	1 [100.00]	5 [100.00]	6 [100.00]	8 [100.00]	9 [100.00]	0 [100.00]	1 [100.00]
1 [62.50]	2 [62.50]	1 [62.50]	2 [62.50]	6 [62.50]	7 [62.50]	3 [83.33]	4 [83.33]	7 [83.33]	8 [83.33]
7 [62.50]	8 [62.50]	2 [62.50]	3 [62.50]	7 [62.50]	8 [62.50]	6 [70.00]	7 [70.00]	10 [70.00]	11 [70.00]
3 [53.33]	4 [53.33]	4 [53.33]	5 [53.33]	3 [53.33]	4 [53.33]	2 [62.50]	3 [62.50]	6 [62.50]	7 [62.50]
4 [53.33]	5 [53.33]	10 [53.33]	11 [53.33]	9 [53.33]	10 [53.33]	4 [33.33]	5 [33.33]	8 [33.33]	9 [33.33]
9 [53.33]	10 [53.33]	3 [33.33]	4 [33.33]	8 [33.33]	9 [33.33]	9 [32.50]	10 [32.50]	1 [32.50]	2 [32.50]
10 [53.33]	11 [53.33]	9 [32.50]	10 [32.50]	2 [32.50]	3 [32.50]	10 [32.50]	11 [32.50]	2 [32.50]	3 [32.50]
2 [12.50]	3 [12.50]	5 [20.00]	6 [20.00]	10 [20.00]	11 [20.00]	5 [20.00]	6 [20.00]	9 [20.00]	10 [20.00]
8 [12.50]	9 [12.50]	8 [12.50]	9 [12.50]	1 [12.50]	2 [12.50]	1 [12.50]	2 [12.50]	5 [12.50]	6 [12.50]

to 220%										
to 210%										
to 200%										
to 190%										
to 180%										
to 170%										
to 160%										
to 150%										
to 140%										
to 130%										
to 120%										
to 110%										
to 100%										
to 90%										
to 80%										
to 70%										
to 60%										
to 50%										
to 40%										
to 30%										
to 20%							0-7-8-11 12.50 %	0-1-8-9 12.50 %	0-3-4-11 12.50 %	0-1-4-5 12.50 %
to 10%										
0% / 2 equal roots	0-5-6-11 0 % 2 equal roots	0-1-6-7 0 % 2 equal roots	0-6-7-11 0 % 2 equal roots	0-1-7-8 0 % 2 equal roots	0-4-5-11 0 % 2 equal roots	0-1-5-6 0 % 2 equal roots				
0% / 3 equal roots										
0% / 4+ equal roots										
Root clarity	TSC4-9		TSC4-8				TSC4-7			

to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

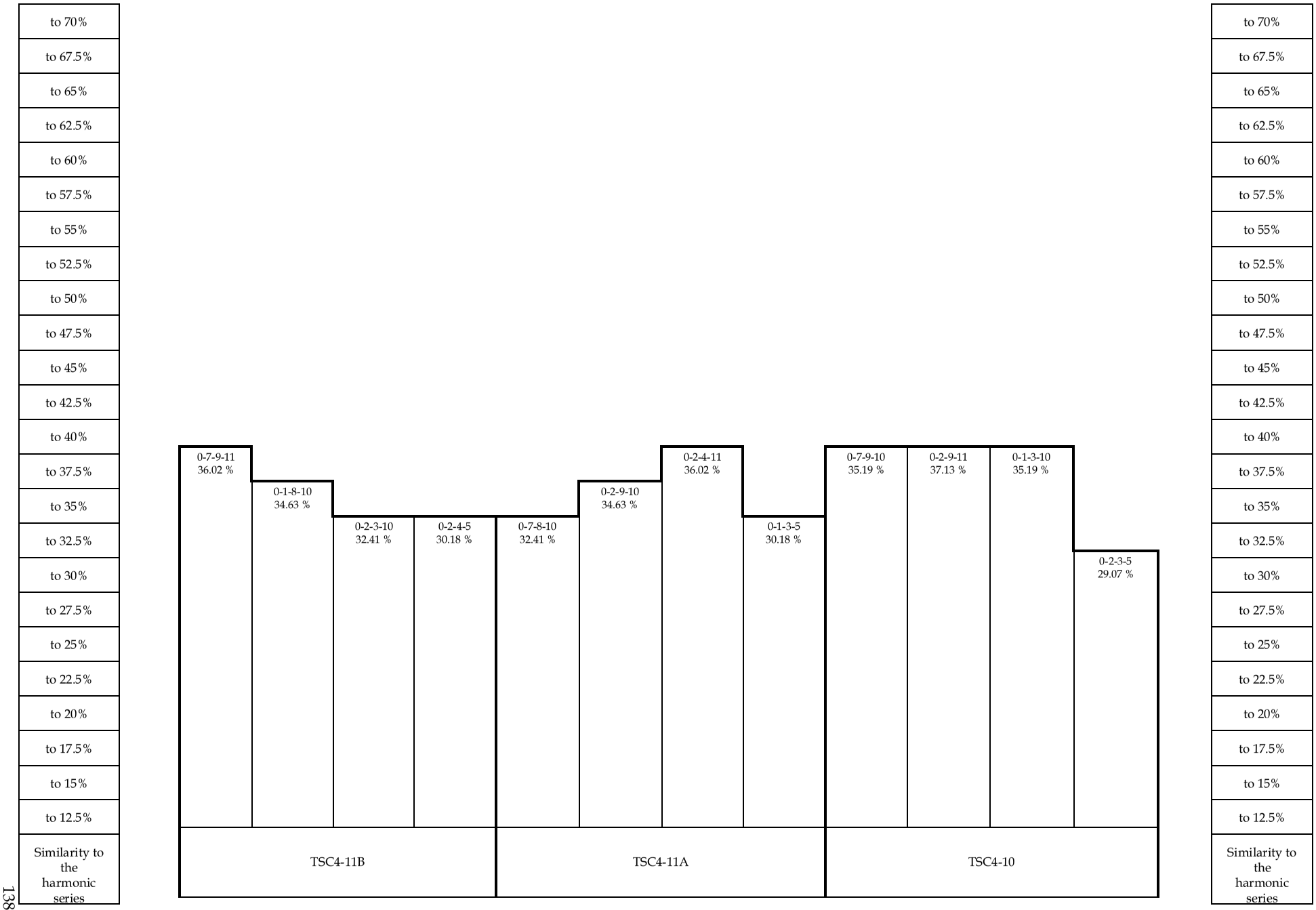
TSC4-9		TSC4-8				TSC4-7			
0-5-6-11	0-1-6-7	0-6-7-11	0-1-7-8	0-4-5-11	0-1-5-6	0-7-8-11	0-1-8-9	0-3-4-11	0-1-4-5
5 [175.00]	0 [175.00]	0 [175.00]	0 [175.00]	4 [175.00]	5 [175.00]	0 [175.00]	1 [175.00]	4 [175.00]	5 [175.00]
11 [175.00]	6 [175.00]	11 [175.00]	1 [175.00]	5 [175.00]	6 [175.00]	7 [155.56]	8 [155.56]	0 [155.56]	0 [155.56]
0 [100.00]	1 [100.00]	7 [155.56]	8 [155.56]	0 [155.56]	1 [155.56]	8 [155.56]	9 [155.56]	11 [155.56]	1 [155.56]
6 [100.00]	7 [100.00]	6 [100.00]	7 [100.00]	11 [100.00]	0 [100.00]	11 [100.00]	0 [100.00]	3 [100.00]	4 [100.00]
4 [72.92]	5 [72.92]	4 [72.92]	5 [72.92]	9 [72.92]	10 [72.92]	4 [97.22]	5 [97.22]	8 [97.22]	9 [97.22]
10 [72.92]	11 [72.92]	5 [72.92]	6 [72.92]	10 [72.92]	11 [72.92]	1 [81.67]	2 [81.67]	5 [81.67]	6 [81.67]
1 [62.22]	2 [62.22]	2 [62.22]	3 [62.22]	1 [62.22]	2 [62.22]	5 [72.92]	6 [72.92]	9 [72.92]	10 [72.92]
2 [62.22]	3 [62.22]	8 [62.22]	9 [62.22]	7 [62.22]	8 [62.22]	9 [37.92]	10 [37.92]	1 [37.92]	2 [37.92]
7 [62.22]	8 [62.22]	9 [37.92]	10 [37.92]	2 [37.92]	3 [37.92]	10 [37.92]	11 [37.92]	2 [37.92]	3 [37.92]
8 [62.22]	9 [62.22]	3 [33.33]	4 [33.33]	8 [33.33]	9 [33.33]	3 [33.33]	4 [33.33]	7 [33.33]	8 [33.33]
3 [12.50]	4 [12.50]	1 [20.00]	2 [20.00]	6 [20.00]	7 [20.00]	2 [20.00]	3 [20.00]	6 [20.00]	7 [20.00]
9 [12.50]	10 [12.50]	10 [12.50]	11 [12.50]	3 [12.50]	4 [12.50]	6 [12.50]	7 [12.50]	10 [12.50]	11 [12.50]

Names

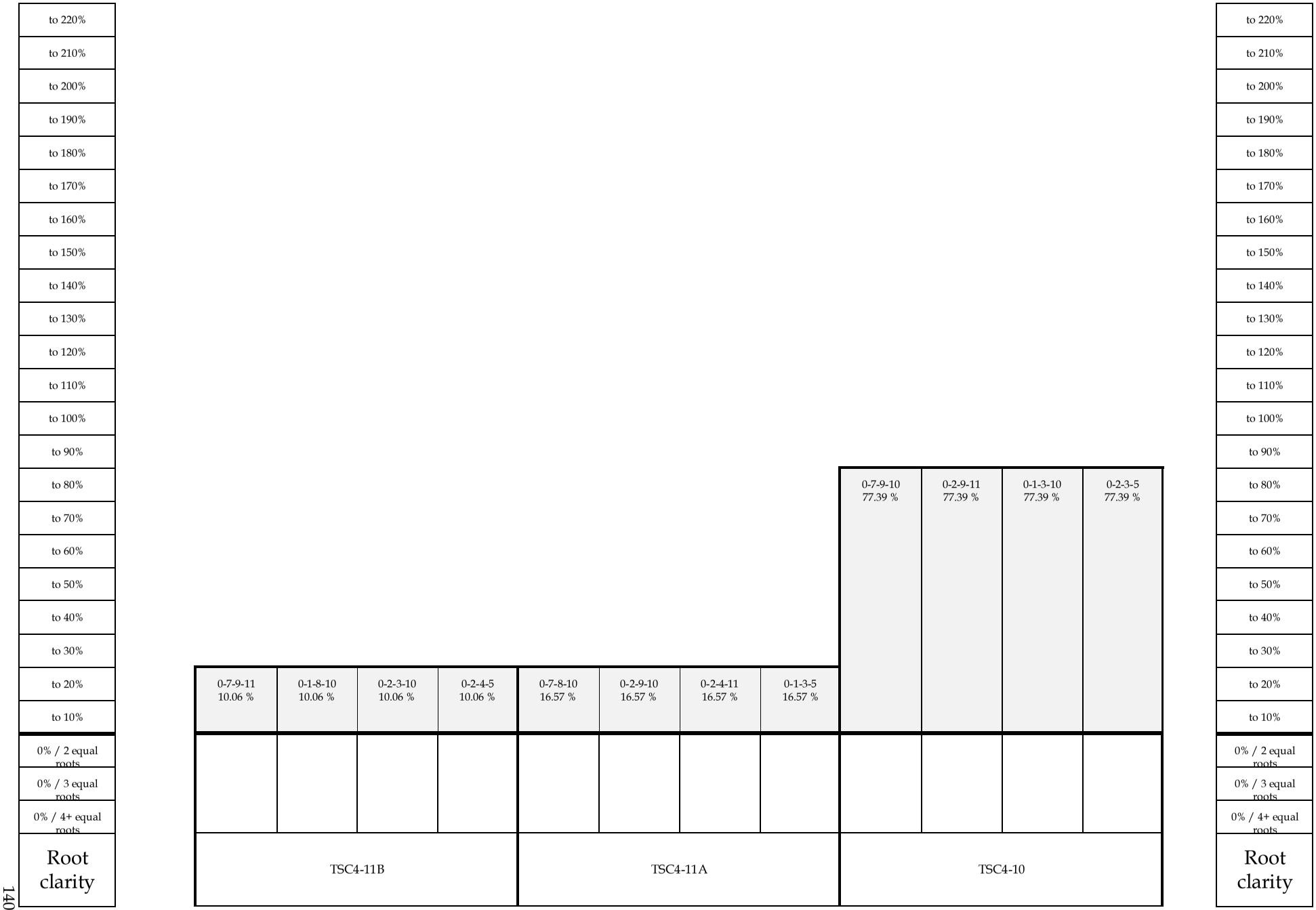
0-5-6-11 Messiaen's Truncated Mode 5 Inverse scale	0-1-6-7 Messiaen's Truncated Mode 5 scale	0-6-7-11 0-6-7-11 tetrachord	0-1-7-8 0-1-7-8 tetrachord	0-4-5-11 0-4-5-11 tetrachord	0-1-5-6 0-1-5-6 tetrachord	0-7-8-11 0-7-8-11 tetrachord	0-1-8-9 0-1-8-9 tetrachord	0-3-4-11 0-3-4-11 tetrachord	0-1-4-5 0-1-4-5 tetrachord		
TSC4-9		TSC4-8				TSC4-7					

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-7-9-11
36.02 %
0-1-8-10
34.63 %
0-2-3-10
32.41 %
0-2-4-5
30.18 %
0-7-8-10
32.41 %
0-2-9-10
34.63 %
0-2-4-11
36.02 %
0-1-3-5
30.18 %
0-7-9-10
35.19 %
0-2-9-11
37.13 %
0-1-3-10
35.19 %
0-2-3-5
29.07 %
TSC4-11B
TSC4-11A
TSC4-10
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-11B				TSC4-11A				TSC4-10			
0-7-9-11	0-1-8-10	0-2-3-10	0-2-4-5	0-7-8-10	0-2-9-10	0-2-4-11	0-1-3-5	0-7-9-10	0-2-9-11	0-1-3-10	0-2-3-5
7 [170.00]	8 [170.00]	10 [170.00]	0 [170.00]	7 [150.00]	9 [150.00]	11 [150.00]	0 [150.00]	7 [170.00]	9 [170.00]	10 [170.00]	0 [170.00]
11 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	5 [145.83]	10 [120.00]	0 [120.00]	1 [120.00]	3 [120.00]
9 [132.50]	10 [132.50]	0 [132.50]	2 [132.50]	10 [132.50]	0 [132.50]	2 [132.50]	3 [132.50]	0 [112.50]	2 [112.50]	0 [112.50]	2 [112.50]
0 [100.00]	1 [100.00]	3 [100.00]	5 [100.00]	8 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]	9 [112.50]	11 [112.50]	3 [112.50]	5 [112.50]
4 [83.33]	5 [83.33]	7 [83.33]	9 [83.33]	2 [95.83]	4 [95.83]	6 [95.83]	7 [95.83]	2 [95.83]	4 [95.83]	5 [95.83]	7 [95.83]
2 [62.50]	3 [62.50]	5 [62.50]	7 [62.50]	5 [70.00]	7 [70.00]	9 [70.00]	10 [70.00]	4 [83.33]	6 [83.33]	7 [83.33]	9 [83.33]
6 [50.00]	7 [50.00]	9 [50.00]	11 [50.00]	3 [50.00]	5 [50.00]	7 [50.00]	8 [50.00]	5 [70.00]	7 [70.00]	8 [70.00]	10 [70.00]
1 [45.83]	2 [45.83]	4 [45.83]	6 [45.83]	4 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	11 [45.83]	1 [45.83]	2 [45.83]	4 [45.83]
3 [33.33]	4 [33.33]	6 [33.33]	8 [33.33]	11 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	6 [33.33]
5 [20.00]	6 [20.00]	1 [20.00]	3 [20.00]	6 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	1 [20.00]
10 [20.00]	11 [20.00]	8 [20.00]	10 [20.00]	9 [12.50]	11 [12.50]	1 [12.50]	2 [12.50]				


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-7-9-11 10.06 %
0-1-8-10 10.06 %
0-2-3-10 10.06 %
0-2-4-5 10.06 %
0-7-8-10 16.57 %
0-2-9-10 16.57 %
0-2-4-11 16.57 %
0-1-3-5 16.57 %
0-7-9-10 77.39 %
0-2-9-11 77.39 %
0-1-3-10 77.39 %
0-2-3-5 77.39 %
TSC4-11B
TSC4-11A
TSC4-10
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

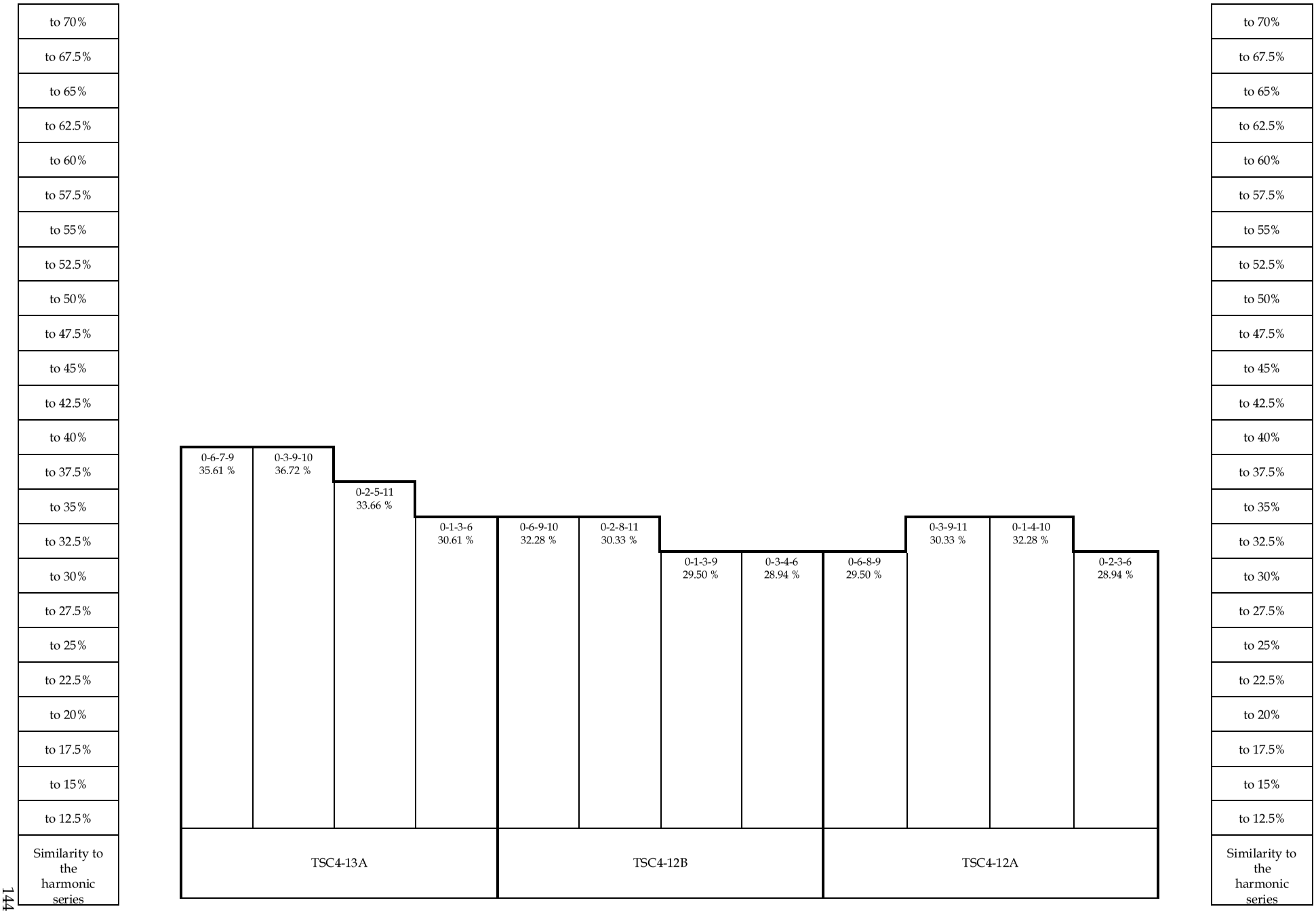
TSC4-11B				TSC4-11A				TSC4-10			
0-7-9-11	0-1-8-10	0-2-3-10	0-2-4-5	0-7-8-10	0-2-9-10	0-2-4-11	0-1-3-5	0-7-9-10	0-2-9-11	0-1-3-10	0-2-3-5
7 [218.75]	8 [218.75]	10 [218.75]	0 [218.75]	0 [255.00]	2 [255.00]	4 [255.00]	5 [255.00]	0 [255.00]	2 [255.00]	3 [255.00]	5 [255.00]
9 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]	8 [218.75]	10 [218.75]	0 [218.75]	1 [218.75]	5 [143.75]	7 [143.75]	8 [143.75]	10 [143.75]
0 [175.00]	1 [175.00]	3 [175.00]	5 [175.00]	10 [198.75]	0 [198.75]	2 [198.75]	3 [198.75]	9 [140.00]	11 [140.00]	0 [140.00]	2 [140.00]
5 [143.75]	6 [143.75]	8 [143.75]	10 [143.75]	7 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]	7 [131.25]	0 [131.25]	1 [131.25]	0 [131.25]
11 [140.00]	0 [140.00]	2 [140.00]	4 [140.00]	3 [97.22]	5 [97.22]	7 [97.22]	8 [97.22]	10 [131.25]	9 [131.25]	10 [131.25]	3 [131.25]
2 [81.67]	3 [81.67]	5 [81.67]	7 [81.67]	5 [72.92]	7 [72.92]	9 [72.92]	10 [72.92]	3 [97.22]	5 [97.22]	6 [97.22]	8 [97.22]
4 [50.00]	5 [50.00]	7 [50.00]	9 [50.00]	6 [53.47]	8 [53.47]	10 [53.47]	11 [53.47]	2 [81.67]	4 [81.67]	5 [81.67]	7 [81.67]
3 [33.33]	4 [33.33]	6 [33.33]	1 [33.33]	1 [50.00]	3 [50.00]	5 [50.00]	6 [50.00]	8 [53.47]	10 [53.47]	11 [53.47]	1 [53.47]
8 [33.33]	9 [33.33]	11 [33.33]	8 [33.33]	4 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]
1 [20.00]	2 [20.00]	4 [20.00]	6 [20.00]	2 [20.00]	4 [20.00]	1 [20.00]	2 [20.00]	11 [20.00]	1 [20.00]	2 [20.00]	4 [20.00]
10 [12.50]	11 [12.50]	1 [12.50]	3 [12.50]	9 [20.00]	11 [20.00]	6 [20.00]	7 [20.00]				

Names

TSC4-11B					TSC4-11A					TSC4-10				
0-7-9-11 0-7-9-11 tetrachord	0-1-8-10 0-1-8-10 tetrachord	0-2-3-10 Warao tetratonic scale (Native South America)	0-2-4-5 0-2-4-5 tetrachord		0-7-8-10 0-7-8-10 tetrachord	0-2-9-10 0-2-9-10 tetrachord	0-2-4-11 0-2-4-11 tetrachord	0-1-3-5 Prime number seventh B		0-7-9-10 0-7-9-10 tetrachord	0-2-9-11 0-2-9-11 tetrachord	0-1-3-10 0-1-3-10 tetrachord	0-2-3-5 Prime number seventh A	

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-6-7-9
35.61 %
0-3-9-10
36.72 %
0-2-5-11
33.66 %
0-1-3-6
30.61 %
0-6-9-10
32.28 %
0-2-8-11
30.33 %
0-1-3-9
29.50 %
0-3-4-6
28.94 %
0-6-8-9
29.50 %
0-3-9-11
30.33 %
0-1-4-10
32.28 %
0-2-3-6
28.94 %
TSC4-13A
TSC4-12B
TSC4-12A
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
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to 32.5%
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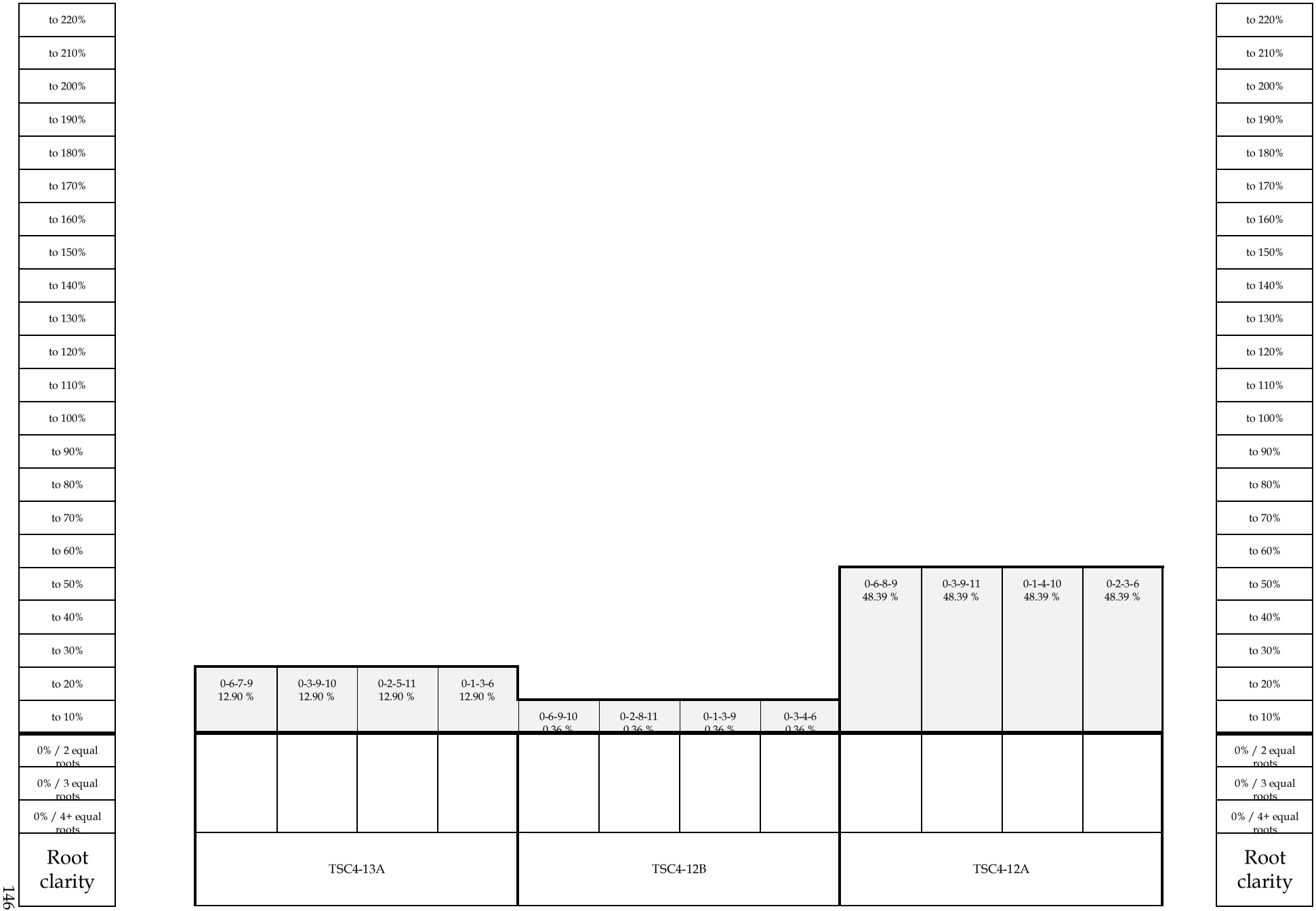
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-13A				TSC4-12B				TSC4-12A			
0-6-7-9	0-3-9-10	0-2-5-11	0-1-3-6	0-6-9-10	0-2-8-11	0-1-3-9	0-3-4-6	0-6-8-9	0-3-9-11	0-1-4-10	0-2-3-6
7 [170.00]	10 [170.00]	0 [170.00]	1 [170.00]	10 [153.33]	0 [153.33]	1 [153.33]	4 [153.33]	0 [133.33]	3 [133.33]	4 [133.33]	6 [133.33]
9 [112.50]	0 [112.50]	2 [112.50]	3 [112.50]	0 [112.50]	2 [112.50]	3 [112.50]	6 [112.50]	6 [120.00]	9 [120.00]	10 [120.00]	0 [120.00]
4 [103.33]	7 [103.33]	9 [103.33]	10 [103.33]	4 [103.33]	6 [103.33]	7 [103.33]	10 [103.33]	8 [112.50]	11 [112.50]	0 [112.50]	2 [112.50]
0 [100.00]	3 [100.00]	5 [100.00]	0 [100.00]	6 [100.00]	8 [100.00]	0 [100.00]	0 [100.00]	4 [103.33]	7 [103.33]	8 [103.33]	10 [103.33]
6 [100.00]	9 [100.00]	11 [100.00]	6 [100.00]	9 [100.00]	11 [100.00]	9 [100.00]	3 [100.00]	9 [100.00]	0 [100.00]	1 [100.00]	3 [100.00]
1 [83.33]	4 [83.33]	6 [83.33]	7 [83.33]	1 [83.33]	3 [83.33]	4 [83.33]	7 [83.33]	1 [83.33]	4 [83.33]	5 [83.33]	7 [83.33]
2 [62.50]	5 [62.50]	7 [62.50]	8 [62.50]	7 [70.00]	9 [70.00]	10 [70.00]	1 [70.00]	7 [70.00]	10 [70.00]	11 [70.00]	1 [70.00]
10 [53.33]	1 [53.33]	3 [53.33]	4 [53.33]	5 [50.00]	7 [50.00]	8 [50.00]	11 [50.00]	10 [65.83]	1 [65.83]	2 [65.83]	4 [65.83]
11 [45.83]	2 [45.83]	4 [45.83]	5 [45.83]	2 [45.83]	4 [45.83]	5 [45.83]	8 [45.83]	3 [50.00]	6 [50.00]	7 [50.00]	9 [50.00]
5 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	8 [32.50]	10 [32.50]	11 [32.50]	2 [32.50]	2 [12.50]	2 [12.50]	3 [12.50]	5 [12.50]
8 [12.50]	11 [12.50]	1 [12.50]	2 [12.50]	11 [12.50]	1 [12.50]	2 [12.50]	5 [12.50]	11 [12.50]	5 [12.50]	6 [12.50]	8 [12.50]


to 220%
to 210%
to 200%
to 190%
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to 170%
to 160%
to 150%
to 140%
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to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-6-7-9 12.90 %
0-3-9-10 12.90 %
0-2-5-11 12.90 %
0-1-3-6 12.90 %
0-6-9-10 0.36 %
0-2-8-11 0.36 %
0-1-3-9 0.36 %
0-3-4-6 0.36 %
0-6-8-9 48.39 %
0-3-9-11 48.39 %
0-1-4-10 48.39 %
0-2-3-6 48.39 %
TSC4-13A
TSC4-12B
TSC4-12A
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
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to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

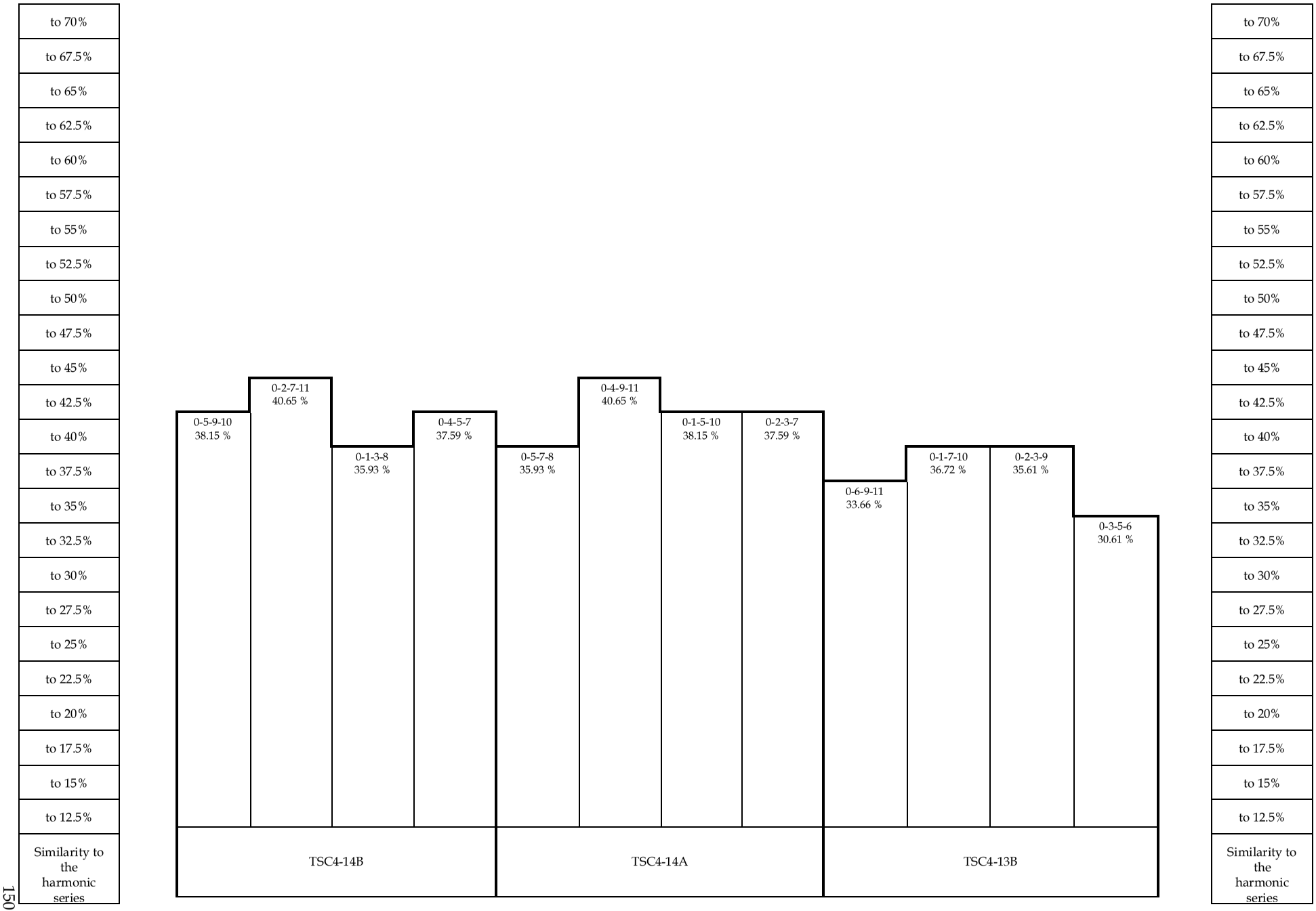
TSC4-13A				TSC4-12B				TSC4-12A			
0-6-7-9	0-3-9-10	0-2-5-11	0-1-3-6	0-6-9-10	0-2-8-11	0-1-3-9	0-3-4-6	0-6-8-9	0-3-9-11	0-1-4-10	0-2-3-6
0 175.00 2 155.00 5 143.75 9 140.00 7 131.25 6 100.00 11 81.67 8 62.22 3 33.33 4 12.50 10 12.50	3 175.00 5 155.00 8 143.75 0 140.00 10 131.25 9 100.00 2 81.67 11 62.22 6 33.33 1 12.50 7 12.50	5 [175.00] 7 [155.00] 10 [143.75] 2 [140.00] 0 [131.25] 11 [100.00] 4 [81.67] 1 [62.22] 8 [33.33] 3 [12.50] 9 [12.50]	6 [175.00] 8 [155.00] 11 [143.75] 3 [140.00] 1 [131.25] 0 [100.00] 5 [81.67] 2 [62.22] 9 [33.33] 4 [12.50] 10 [12.50]	6 [155.56] 2 [155.00] 0 [140.00] 10 [131.25] 9 [100.00] 8 [98.75] 5 [97.22] 11 [81.67] 3 [50.00] 4 [12.50] 7 [12.50]	8 [155.56] 4 [155.00] 2 [140.00] 0 [131.25] 11 [100.00] 10 [98.75] 7 [97.22] 1 [81.67] 5 [50.00] 6 [12.50] 9 [12.50]	9 [155.56] 5 [155.00] 3 [140.00] 1 [131.25] 0 [100.00] 11 [98.75] 8 [97.22] 2 [81.67] 6 [50.00] 7 [12.50] 10 [12.50]	0 [155.56] 8 [155.00] 6 [140.00] 4 [131.25] 3 [100.00] 2 [98.75] 11 [97.22] 5 [81.67] 9 [50.00] 1 [12.50] 10 [12.50]	8 [230.00] 2 [155.00] 6 [131.25] 0 [100.00] 9 [100.00] 5 [97.22] 11 [81.67] 4 [53.47] 1 [50.00] 10 [37.92] 7 [12.50]	11 [230.00] 5 [155.00] 9 [131.25] 0 [100.00] 3 [100.00] 8 [97.22] 2 [81.67] 7 [53.47] 4 [50.00] 1 [37.92] 10 [12.50]	0 [230.00] 6 [155.00] 10 [131.25] 1 [100.00] 4 [100.00] 9 [97.22] 3 [81.67] 8 [53.47] 5 [50.00] 2 [37.92] 11 [12.50]	2 [230.00] 8 [155.00] 0 [131.25] 3 [100.00] 6 [100.00] 11 [97.22] 5 [81.67] 10 [53.47] 7 [50.00] 4 [37.92] 1 [12.50]

Names

0-6-8-9 0-6-8-9 tetrachord	0-3-9-11 0-3-9-11 tetrachord	0-1-4-10 0-1-4-10 tetrachord	0-2-3-6 0-2-3-6 tetrachord		TSC4-12A
0-6-9-10 0-6-9-10 tetrachord	0-2-8-11 0-2-8-11 tetrachord	0-1-3-9 0-1-3-9 tetrachord	0-3-4-6 0-3-4-6 tetrachord		TSC4-12B
0-6-7-9 0-6-7-9 tetrachord	0-3-9-10 0-3-9-10 tetrachord	0-2-5-11 0-2-5-11 tetrachord	0-1-3-6 0-1-3-6 tetrachord		TSC4-13A

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
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to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-9-10
38.15 %
0-2-7-11
40.65 %
0-1-3-8
35.93 %
0-4-5-7
37.59 %
0-5-7-8
35.93 %
0-4-9-11
40.65 %
0-1-5-10
38.15 %
0-2-3-7
37.59 %
0-6-9-11
33.66 %
0-1-7-10
36.72 %
0-2-3-9
35.61 %
0-3-5-6
30.61 %
TSC4-14B
TSC4-14A
TSC4-13B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
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to 47.5%
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to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-14B				TSC4-14A				TSC4-13B			
0-5-9-10	0-2-7-11	0-1-3-8	0-4-5-7	0-5-7-8	0-4-9-11	0-1-5-10	0-2-3-7	0-6-9-11	0-1-7-10	0-2-3-9	0-3-5-6
0 [162.50]	2 [162.50]	3 [162.50]	7 [162.50]	0 [183.33]	4 [183.33]	5 [183.33]	7 [183.33]	6 [150.00]	7 [150.00]	9 [150.00]	0 [150.00]
5 [150.00]	7 [150.00]	8 [150.00]	0 [150.00]	7 [162.50]	11 [162.50]	0 [162.50]	2 [162.50]	9 [120.00]	10 [120.00]	0 [120.00]	3 [120.00]
9 [133.33]	11 [133.33]	0 [133.33]	4 [133.33]	5 [120.00]	9 [120.00]	10 [120.00]	0 [120.00]	11 [112.50]	0 [112.50]	2 [112.50]	5 [112.50]
10 [120.00]	0 [120.00]	1 [120.00]	5 [120.00]	8 [100.00]	0 [100.00]	1 [100.00]	3 [100.00]	4 [103.33]	5 [103.33]	7 [103.33]	10 [103.33]
4 [83.33]	6 [83.33]	7 [83.33]	11 [83.33]	3 [70.00]	7 [70.00]	8 [70.00]	10 [70.00]	0 [100.00]	1 [100.00]	3 [100.00]	6 [100.00]
7 [82.50]	9 [82.50]	10 [82.50]	2 [82.50]	2 [62.50]	6 [62.50]	7 [62.50]	9 [62.50]	1 [95.83]	2 [95.83]	4 [95.83]	7 [95.83]
2 [45.83]	4 [45.83]	5 [45.83]	9 [45.83]	9 [45.83]	1 [45.83]	2 [45.83]	4 [45.83]	7 [70.00]	8 [70.00]	10 [70.00]	1 [70.00]
1 [33.33]	3 [33.33]	4 [33.33]	8 [33.33]	4 [33.33]	3 [33.33]	4 [33.33]	6 [33.33]	10 [53.33]	11 [53.33]	1 [53.33]	4 [53.33]
3 [20.00]	5 [20.00]	6 [20.00]	3 [20.00]	11 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]	3 [33.33]	4 [33.33]	6 [33.33]	9 [33.33]
8 [20.00]	10 [20.00]	11 [20.00]	10 [20.00]	10 [32.50]	2 [32.50]	3 [32.50]	5 [32.50]	2 [12.50]	3 [12.50]	5 [12.50]	2 [12.50]
11 [12.50]	1 [12.50]	2 [12.50]	6 [12.50]	6 [20.00]	10 [20.00]	11 [20.00]	1 [20.00]	8 [12.50]	9 [12.50]	11 [12.50]	8 [12.50]

to 220%
to 210%
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to 180%
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to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-9-10 12.82 %
0-2-7-11 12.82 %
0-1-3-8 12.82 %
0-4-5-7 12.82 %
0-5-7-8 39.29 %
0-4-9-11 39.29 %
0-1-5-10 39.29 %
0-2-3-7 39.29 %
0-6-9-11 64.52 %
0-1-7-10 64.52 %
0-2-3-9 64.52 %
0-3-5-6 64.52 %
TSC4-14B
TSC4-14A
TSC4-13B
Root clarity

TSC4-14B				TSC4-14A				TSC4-13B			
0-5-9-10	0-2-7-11	0-1-3-8	0-4-5-7	0-5-7-8	0-4-9-11	0-1-5-10	0-2-3-7	0-6-9-11	0-1-7-10	0-2-3-9	0-3-5-6
5 [275.00]	7 [275.00]	8 [275.00]	0 [275.00]	5 [243.75]	9 [243.75]	10 [243.75]	0 [243.75]	11 [255.00]	0 [255.00]	2 [255.00]	5 [255.00]
10 [243.75]	0 [243.75]	1 [243.75]	5 [243.75]	0 [175.00]	4 [175.00]	5 [175.00]	7 [175.00]	2 [155.00]	3 [155.00]	5 [155.00]	8 [155.00]
0 [140.00]	2 [140.00]	3 [140.00]	7 [140.00]	8 [155.56]	0 [155.56]	1 [155.56]	3 [155.56]	9 [131.25]	10 [131.25]	0 [131.25]	3 [131.25]
9 [100.00]	11 [100.00]	0 [100.00]	4 [100.00]	7 [140.00]	11 [140.00]	0 [140.00]	2 [140.00]	0 [100.00]	1 [100.00]	3 [100.00]	0 [100.00]
2 [81.67]	4 [81.67]	5 [81.67]	9 [81.67]	10 [123.75]	2 [123.75]	3 [123.75]	5 [123.75]	6 [100.00]	7 [100.00]	9 [100.00]	6 [100.00]
3 [72.92]	5 [72.92]	6 [72.92]	10 [72.92]	1 [97.22]	5 [97.22]	6 [97.22]	8 [97.22]	5 [97.22]	6 [97.22]	8 [97.22]	11 [97.22]
8 [53.47]	10 [53.47]	11 [53.47]	3 [53.47]	3 [53.47]	7 [53.47]	8 [53.47]	10 [53.47]	4 [72.92]	5 [72.92]	7 [72.92]	10 [72.92]
7 [37.92]	9 [37.92]	10 [37.92]	2 [37.92]	4 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]	8 [62.22]	9 [62.22]	11 [62.22]	2 [62.22]
1 [33.33]	3 [33.33]	4 [33.33]	1 [33.33]	2 [20.00]	1 [20.00]	2 [20.00]	4 [20.00]	7 [53.47]	8 [53.47]	10 [53.47]	1 [53.47]
6 [33.33]	8 [33.33]	9 [33.33]	8 [33.33]	9 [20.00]	6 [20.00]	7 [20.00]	9 [20.00]	1 [20.00]	2 [20.00]	4 [20.00]	7 [20.00]
11 [20.00]	1 [20.00]	2 [20.00]	6 [20.00]	6 [12.50]	10 [12.50]	11 [12.50]	1 [12.50]	10 [12.50]	11 [12.50]	1 [12.50]	4 [12.50]

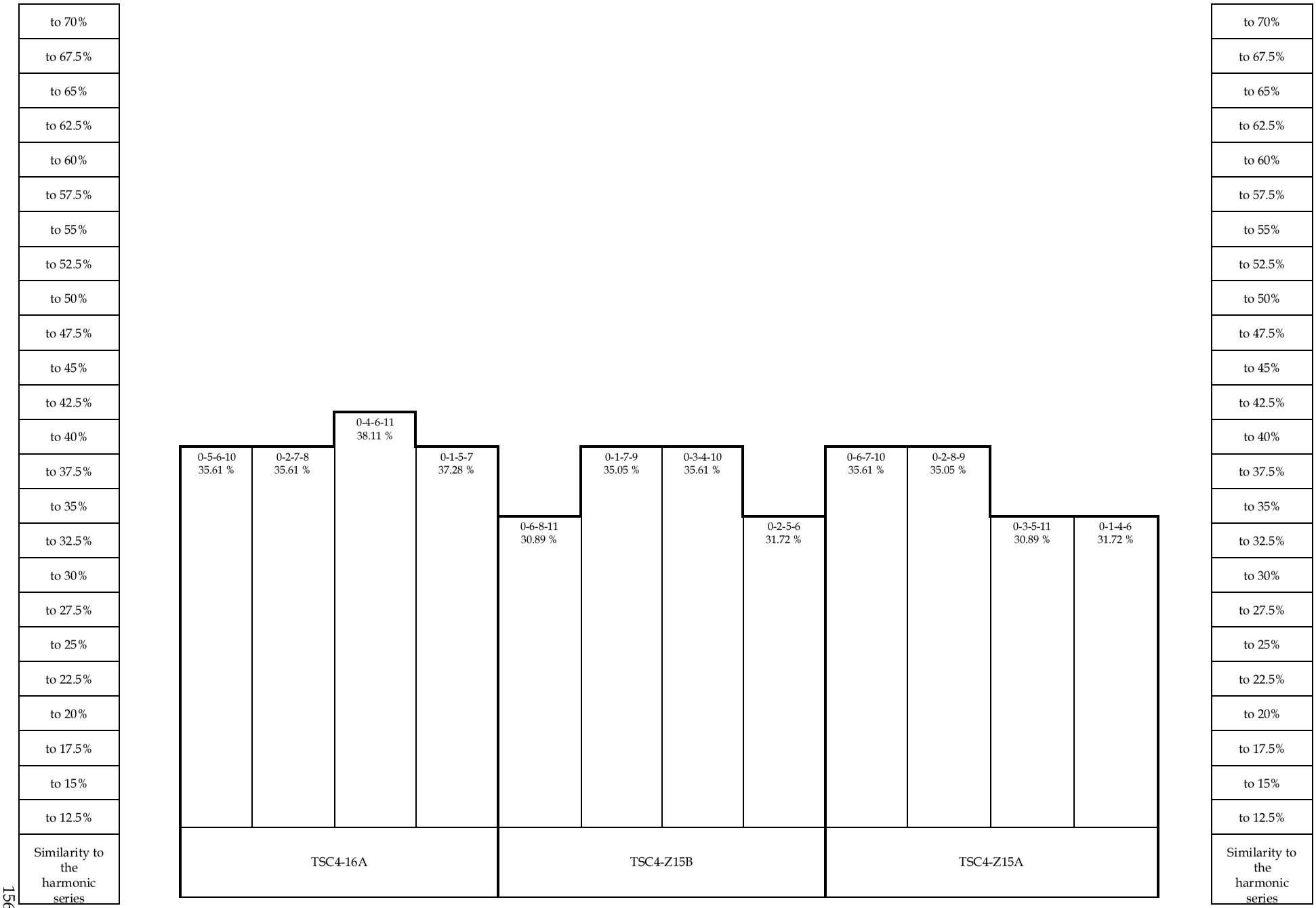
Root candidates and weights

Names

0-5-9-10 Third inversion major add 4 tetrachord	0-2-7-11 Second inversion major add 4 tetrachord	0-1-3-8 First inversion major add 4 tetrachord	0-4-5-7 Major add 4 tetrachord	TSC4-14B
0-5-7-8 Third inversion minor add 2 tetrachord	0-4-9-11 Second inversion minor add 2 tetrachord	0-1-5-10 First inversion minor add 2 tetrachord	0-2-3-7 Minor add 2 tetrachord	TSC4-14A
0-6-9-11 0-6-9-11 tetrachord	0-1-7-10 Mistuned Historical Church Bell 7	0-2-3-9 0-2-3-9 tetrachord	0-3-5-6 0-3-5-6 tetrachord	TSC4-13B

Names

Appendix A continues on the next page.


to 70%
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to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-6-10
35.61 %
0-2-7-8
35.61 %
0-4-6-11
38.11 %
0-1-5-7
37.28 %
0-6-8-11
30.89 %
0-1-7-9
35.05 %
0-3-4-10
35.61 %
0-2-5-6
31.72 %
0-6-7-10
35.61 %
0-2-8-9
35.05 %
0-3-5-11
30.89 %
0-1-4-6
31.72 %
TSC4-16A
TSC4-Z15B
TSC4-Z15A
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-16A				TSC4-Z15B				TSC4-Z15A			
0-5-6-10	0-2-7-8	0-4-6-11	0-1-5-7	0-6-8-11	0-1-7-9	0-3-4-10	0-2-5-6	0-6-7-10	0-2-8-9	0-3-5-11	0-1-4-6
0 [162.50]	2 [162.50]	6 [162.50]	7 [162.50]	6 [170.00]	7 [170.00]	10 [170.00]	0 [170.00]	10 [153.33]	0 [153.33]	3 [153.33]	4 [153.33]
10 [153.33]	0 [153.33]	4 [153.33]	5 [153.33]	0 [133.33]	1 [133.33]	4 [133.33]	6 [133.33]	7 [150.00]	9 [150.00]	0 [150.00]	1 [150.00]
5 [150.00]	7 [150.00]	11 [150.00]	0 [150.00]	8 [112.50]	9 [112.50]	0 [112.50]	2 [112.50]	0 [112.50]	2 [112.50]	5 [112.50]	6 [112.50]
6 [100.00]	8 [100.00]	0 [100.00]	1 [100.00]	11 [100.00]	0 [100.00]	3 [100.00]	5 [100.00]	6 [100.00]	8 [100.00]	11 [100.00]	0 [100.00]
7 [62.50]	9 [62.50]	1 [62.50]	2 [62.50]	3 [83.33]	4 [83.33]	7 [83.33]	9 [83.33]	2 [95.83]	4 [95.83]	7 [95.83]	8 [95.83]
4 [53.33]	6 [53.33]	10 [53.33]	11 [53.33]	10 [65.83]	11 [65.83]	2 [65.83]	4 [65.83]	5 [70.00]	7 [70.00]	10 [70.00]	11 [70.00]
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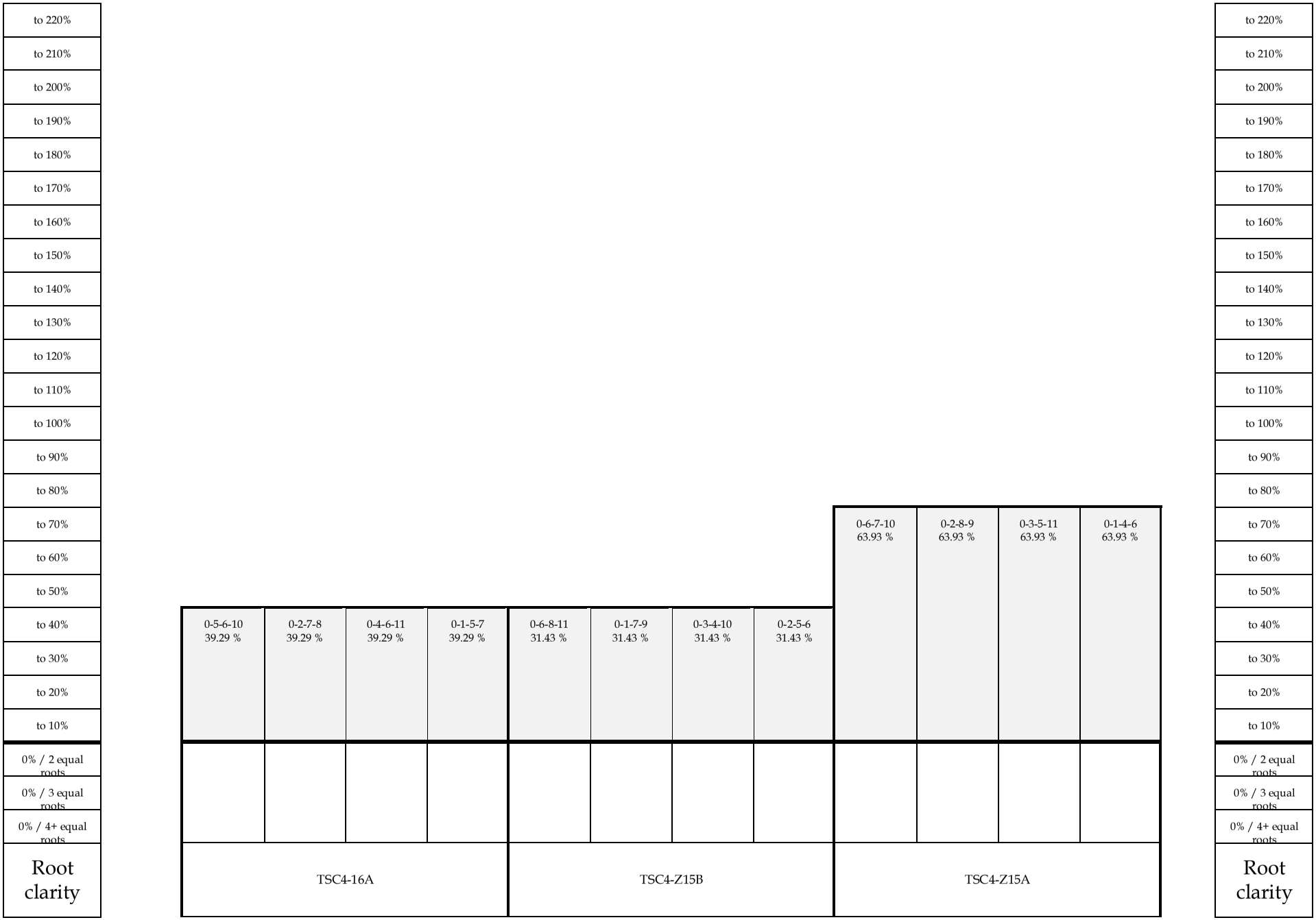
2 [45.83]	4 [45.83]	8 [45.83]	9 [45.83]	4 [53.33]	5 [53.33]	8 [53.33]	10 [53.33]	1 [50.00]	3 [50.00]	6 [50.00]	7 [50.00]
9 [33.33]	11 [33.33]	3 [33.33]	4 [33.33]	7 [50.00]	8 [50.00]	11 [50.00]	1 [50.00]	11 [33.33]	1 [33.33]	4 [33.33]	5 [33.33]
8 [32.50]	10 [32.50]	2 [32.50]	3 [32.50]	9 [20.00]	10 [20.00]	1 [20.00]	3 [20.00]	8 [32.50]	10 [32.50]	1 [32.50]	2 [32.50]
3 [20.00]	5 [20.00]	9 [20.00]	10 [20.00]	2 [12.50]	3 [12.50]	6 [12.50]	8 [12.50]	9 [12.50]	11 [12.50]	2 [12.50]	3 [12.50]


to 220%
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to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-6-10 39.29 %
0-2-7-8 39.29 %
0-4-6-11 39.29 %
0-1-5-7 39.29 %
0-6-8-11 31.43 %
0-1-7-9 31.43 %
0-3-4-10 31.43 %
0-2-5-6 31.43 %
0-6-7-10 63.93 %
0-2-8-9 63.93 %
0-3-5-11 63.93 %
0-1-4-6 63.93 %
TSC4-16A
TSC4-Z15B
TSC4-Z15A
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

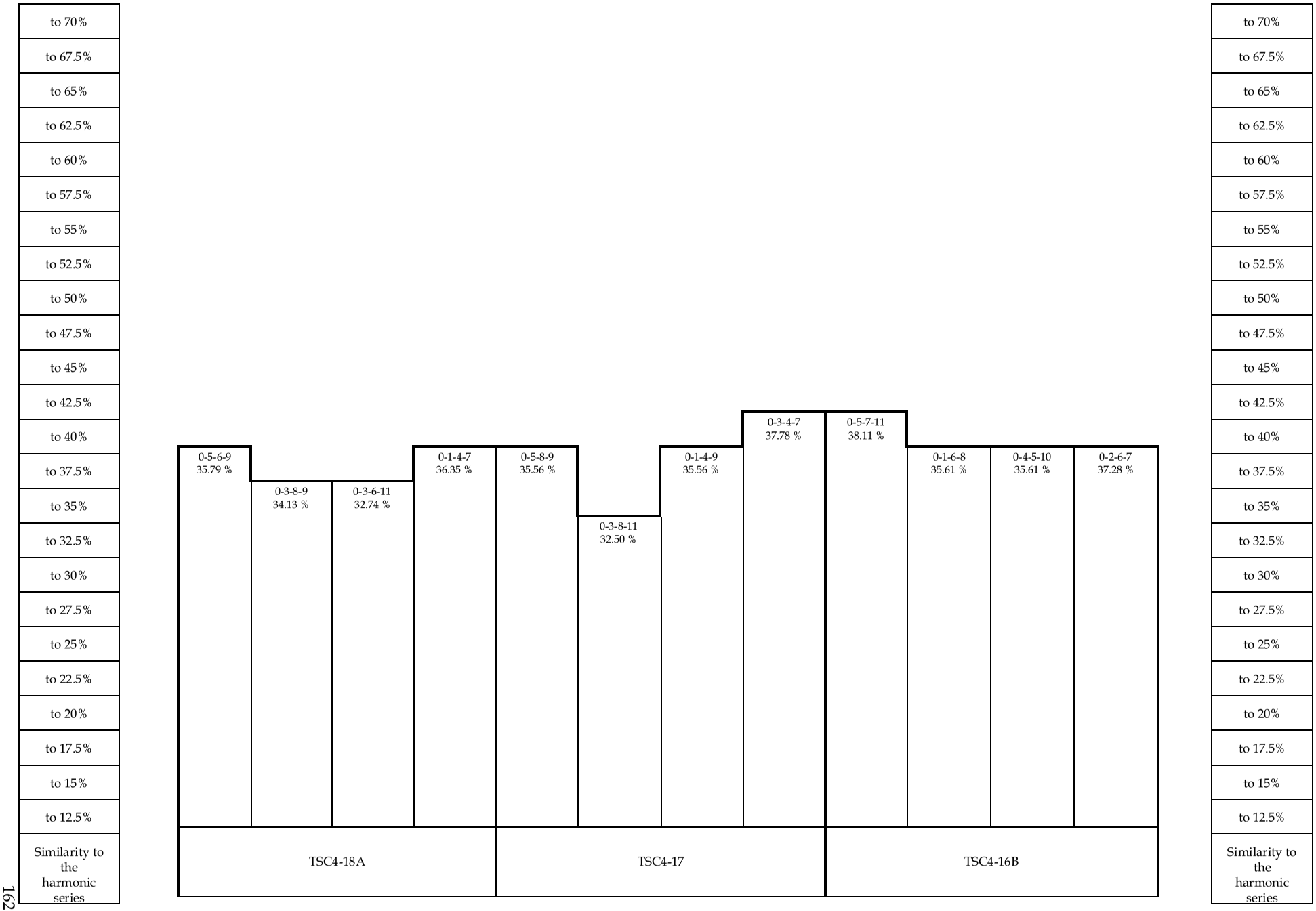
TSC4-16A				TSC4-Z15B				TSC4-Z15A			
0-5-6-10	0-2-7-8	0-4-6-11	0-1-5-7	0-6-8-11	0-1-7-9	0-3-4-10	0-2-5-6	0-6-7-10	0-2-8-9	0-3-5-11	0-1-4-6
10 [243.75] 5 [175.00]	0 [243.75] 7 [175.00]	4 [243.75] 11 [175.00]	5 [243.75] 0 [175.00]	8 [230.00] 11 [175.00]	9 [230.00] 0 [175.00]	0 [230.00] 3 [175.00]	2 [230.00] 5 [175.00]	0 [255.00] 6 [155.56]	2 [255.00] 8 [155.56]	5 [255.00] 11 [155.56]	6 [255.00] 0 [155.56]
6 [155.56] 0 [140.00]	8 [155.56] 2 [140.00]	0 [155.56] 6 [140.00]	1 [155.56] 7 [140.00]	4 [143.75] 6 [131.25]	5 [143.75] 7 [131.25]	8 [143.75] 10 [131.25]	10 [143.75] 0 [131.25]	10 [131.25] 7 [100.00]	0 [131.25] 9 [100.00]	3 [131.25] 0 [100.00]	4 [131.25] 1 [100.00]
8 [98.75] 3 [72.92]	10 [98.75] 5 [72.92]	2 [98.75] 9 [72.92]	3 [98.75] 10 [72.92]	0 [100.00] 1 [81.67]	1 [100.00] 2 [81.67]	4 [100.00] 5 [81.67]	6 [100.00] 7 [81.67]	8 [98.75] 3 [97.22]	10 [98.75] 5 [97.22]	1 [98.75] 8 [97.22]	2 [98.75] 9 [97.22]
2 [62.22] 11 [50.00]	4 [62.22] 1 [50.00]	8 [62.22] 5 [50.00]	9 [62.22] 6 [50.00]	2 [62.22] 5 [50.00]	3 [62.22] 6 [50.00]	6 [62.22] 9 [50.00]	8 [62.22] 11 [50.00]	5 [72.92] 2 [62.22]	7 [72.92] 4 [62.22]	10 [72.92] 7 [62.22]	11 [72.92] 8 [62.22]
1 [33.33] 7 [20.00]	3 [33.33] 9 [20.00]	7 [33.33] 1 [20.00]	8 [33.33] 2 [20.00]	10 [37.92] 7 [33.33]	11 [37.92] 8 [33.33]	2 [37.92] 11 [33.33]	4 [37.92] 1 [33.33]	11 [50.00] 9 [20.00]	1 [50.00] 11 [20.00]	4 [50.00] 2 [20.00]	5 [50.00] 3 [20.00]
4 [12.50]	6 [12.50]	10 [12.50]	11 [12.50]	9 [12.50]	10 [12.50]	1 [12.50]	3 [12.50]	4 [12.50]	6 [12.50]	9 [12.50]	10 [12.50]

Names

0-5-6-10 0-5-6-10 tetrachord	0-2-7-8 0-2-7-8 tetrachord	0-4-6-11 Major seventh plus four tetrachord	0-1-5-7 0-1-5-7 tetrachord	0-6-8-11 0-6-8-11 tetrachord	0-1-7-9 0-1-7-9 tetrachord	0-3-4-10 0-3-4-10 tetrachord	0-2-5-6 0-2-5-6 tetrachord	0-6-7-10 0-6-7-10 tetrachord	0-2-8-9 0-2-8-9 tetrachord	0-3-5-11 0-3-5-11 tetrachord	0-1-4-6 All-interval tetrachord A
TSC4-16A				TSC4-Z15B				TSC4-Z15A			

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
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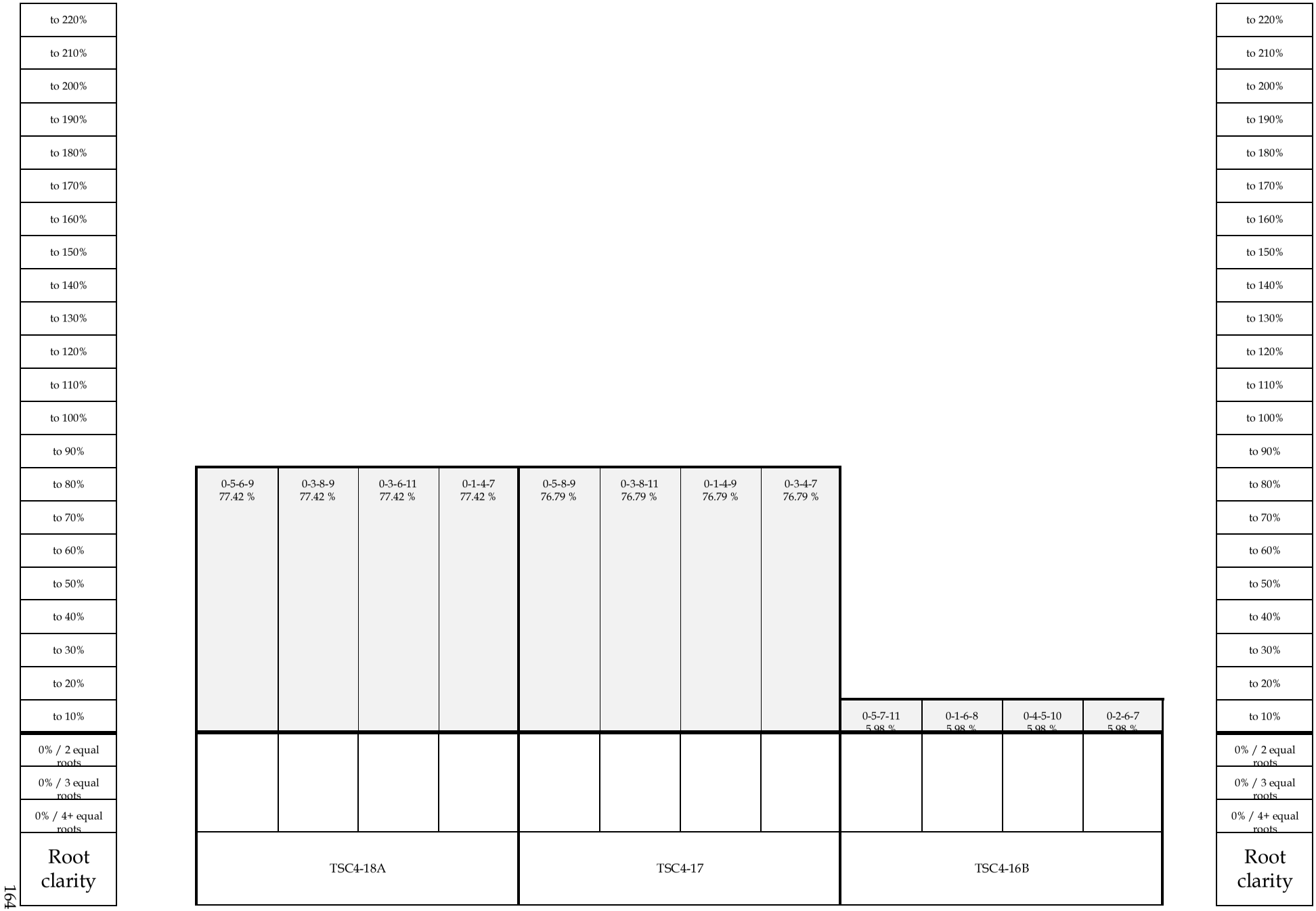
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-6-9
35.79 %
0-3-8-9
34.13 %
0-3-6-11
32.74 %
0-1-4-7
36.35 %
0-5-8-9
35.56 %
0-3-8-11
32.50 %
0-1-4-9
35.56 %
0-3-4-7
37.78 %
0-5-7-11
38.11 %
0-1-6-8
35.61 %
0-4-5-10
35.61 %
0-2-6-7
37.28 %
TSC4-18A
TSC4-17
TSC4-16B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-18A				TSC4-17				TSC4-16B			
0-5-6-9	0-3-8-9	0-3-6-11	0-1-4-7	0-5-8-9	0-3-8-11	0-1-4-9	0-3-4-7	0-5-7-11	0-1-6-8	0-4-5-10	0-2-6-7
0 [150.00]	3 [150.00]	6 [150.00]	7 [150.00]	0 [183.33]	3 [183.33]	4 [183.33]	7 [183.33]	7 [162.50]	8 [162.50]	0 [162.50]	2 [162.50]
9 [133.33]	0 [133.33]	3 [133.33]	4 [133.33]	9 [133.33]	0 [133.33]	1 [133.33]	4 [133.33]	0 [150.00]	1 [150.00]	5 [150.00]	7 [150.00]
4 [103.33]	7 [103.33]	10 [103.33]	11 [103.33]	5 [100.00]	8 [100.00]	0 [100.00]	0 [100.00]	11 [133.33]	0 [133.33]	4 [133.33]	6 [133.33]
5 [100.00]	8 [100.00]	0 [100.00]	0 [100.00]	8 [100.00]	11 [100.00]	9 [100.00]	3 [100.00]	5 [120.00]	6 [120.00]	10 [120.00]	0 [120.00]
6 [100.00]	9 [100.00]	11 [100.00]	1 [100.00]	4 [83.33]	7 [83.33]	8 [83.33]	11 [83.33]	9 [65.83]	10 [65.83]	2 [65.83]	4 [65.83]
1 [83.33]	4 [83.33]	7 [83.33]	8 [83.33]	7 [82.50]	10 [82.50]	11 [82.50]	2 [82.50]	2 [62.50]	3 [62.50]	7 [62.50]	9 [62.50]
7 [82.50]	10 [82.50]	1 [82.50]	2 [82.50]	3 [70.00]	6 [70.00]	7 [70.00]	10 [70.00]	3 [53.33]	4 [53.33]	8 [53.33]	10 [53.33]
10 [53.33]	1 [53.33]	4 [53.33]	5 [53.33]	1 [33.33]	4 [33.33]	5 [33.33]	8 [33.33]	6 [50.00]	7 [50.00]	11 [50.00]	1 [50.00]
3 [20.00]	6 [20.00]	9 [20.00]	10 [20.00]	10 [32.50]	1 [32.50]	2 [32.50]	5 [32.50]	4 [33.33]	5 [33.33]	9 [33.33]	11 [33.33]
2 [12.50]	2 [12.50]	2 [12.50]	3 [12.50]	6 [20.00]	9 [20.00]	10 [20.00]	1 [20.00]	10 [20.00]	11 [20.00]	3 [20.00]	5 [20.00]
8 [12.50]	5 [12.50]	5 [12.50]	6 [12.50]	2 [12.50]	2 [12.50]	3 [12.50]	6 [12.50]	1 [12.50]	2 [12.50]	6 [12.50]	8 [12.50]
11 [12.50]	11 [12.50]	8 [12.50]	9 [12.50]	11 [12.50]	5 [12.50]	6 [12.50]	9 [12.50]				


to 220%
to 210%
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to 160%
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to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-6-9 77.42 %
0-3-8-9 77.42 %
0-3-6-11 77.42 %
0-1-4-7 77.42 %
0-5-8-9 76.79 %
0-3-8-11 76.79 %
0-1-4-9 76.79 %
0-3-4-7 76.79 %
0-5-7-11 5.98 %
0-1-6-8 5.98 %
0-4-5-10 5.98 %
0-2-6-7 5.98 %
TSC4-18A
TSC4-17
TSC4-16B
Root clarity

TSC4-18A				TSC4-17				TSC4-16B			
0-5-6-9	0-3-8-9	0-3-6-11	0-1-4-7	0-5-8-9	0-3-8-11	0-1-4-9	0-3-4-7	0-5-7-11	0-1-6-8	0-4-5-10	0-2-6-7
5 [275.00]	8 [275.00]	11 [275.00]	0 [275.00]	5 [275.00]	8 [275.00]	9 [275.00]	0 [275.00]	5 [243.75]	6 [243.75]	10 [243.75]	0 [243.75]
2 [155.00]	5 [155.00]	8 [155.00]	9 [155.00]	8 [155.56]	11 [155.56]	0 [155.56]	3 [155.56]	7 [230.00]	8 [230.00]	0 [230.00]	2 [230.00]
0 [100.00]	0 [100.00]	0 [100.00]	1 [100.00]	10 [123.75]	1 [123.75]	2 [123.75]	5 [123.75]	0 [175.00]	1 [175.00]	5 [175.00]	7 [175.00]
6 [100.00]	3 [100.00]	3 [100.00]	4 [100.00]	0 [100.00]	0 [100.00]	1 [100.00]	4 [100.00]	11 [100.00]	0 [100.00]	4 [100.00]	6 [100.00]
9 [100.00]	9 [100.00]	6 [100.00]	7 [100.00]	9 [100.00]	3 [100.00]	4 [100.00]	7 [100.00]	10 [72.92]	11 [72.92]	3 [72.92]	5 [72.92]
11 [81.67]	2 [81.67]	5 [81.67]	6 [81.67]	1 [97.22]	4 [97.22]	5 [97.22]	8 [97.22]	1 [62.22]	2 [62.22]	6 [62.22]	8 [62.22]
10 [72.92]	1 [72.92]	4 [72.92]	5 [72.92]	2 [81.67]	5 [81.67]	6 [81.67]	9 [81.67]	3 [53.47]	4 [53.47]	8 [53.47]	10 [53.47]
8 [62.22]	11 [62.22]	2 [62.22]	3 [62.22]	7 [37.92]	10 [37.92]	11 [37.92]	2 [37.92]	4 [50.00]	5 [50.00]	9 [50.00]	11 [50.00]
7 [37.92]	10 [37.92]	1 [37.92]	2 [37.92]	4 [33.33]	7 [33.33]	8 [33.33]	11 [33.33]	9 [37.92]	10 [37.92]	2 [37.92]	4 [37.92]
1 [33.33]	4 [33.33]	7 [33.33]	8 [33.33]	11 [20.00]	2 [20.00]	3 [20.00]	6 [20.00]	8 [33.33]	9 [33.33]	1 [33.33]	3 [33.33]
3 [12.50]	6 [12.50]	9 [12.50]	10 [12.50]	3 [12.50]	6 [12.50]	7 [12.50]	1 [12.50]	2 [20.00]	3 [20.00]	7 [20.00]	9 [20.00]
4 [12.50]	7 [12.50]	10 [12.50]	11 [12.50]	6 [12.50]	9 [12.50]	10 [12.50]	10 [12.50]				

Names

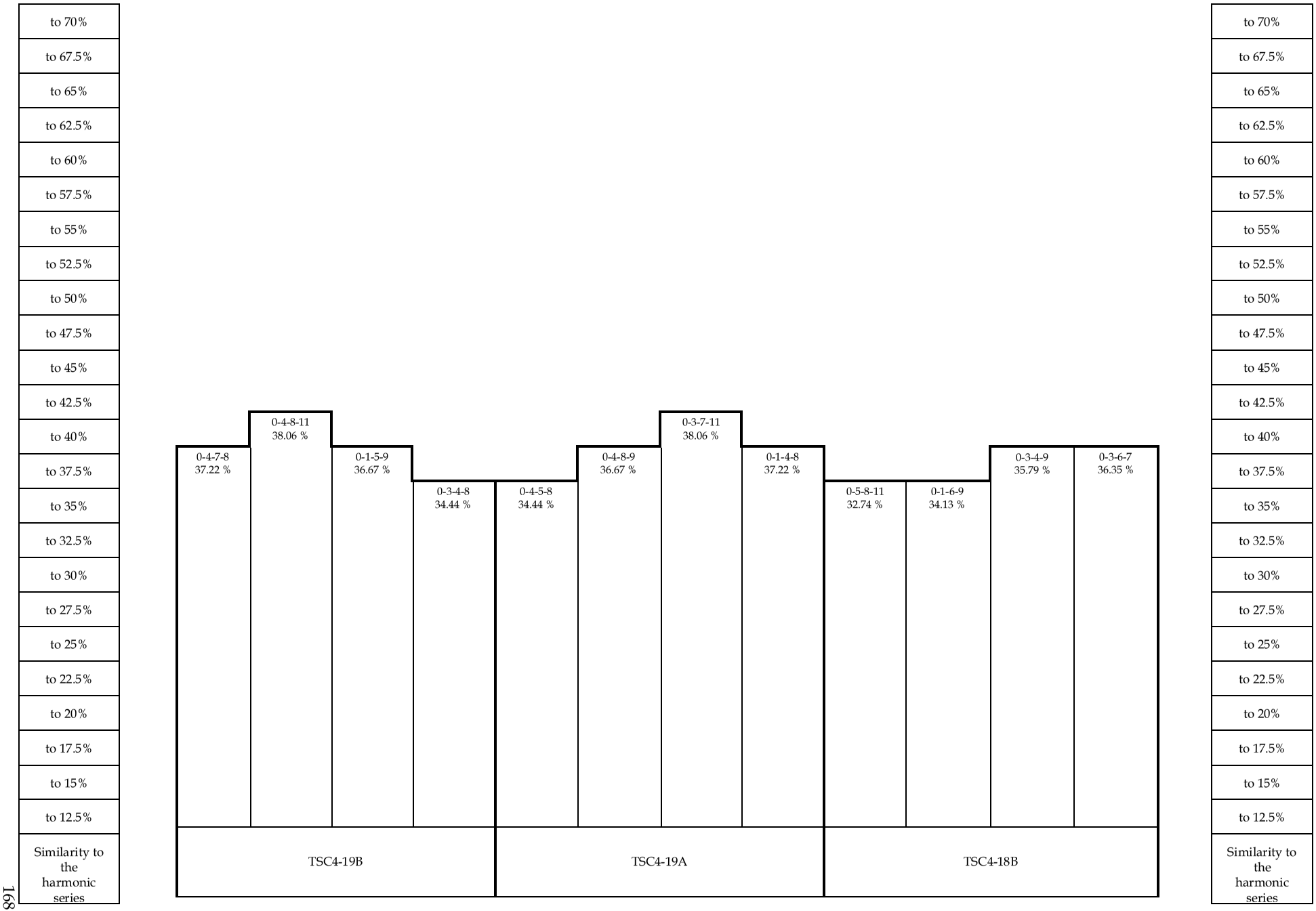
0-5-6-9 Third inversion diminished M7 chord	0-3-8-9 Second inversion diminished M7 chord	0-3-6-11 First inversion diminished M7 chord	0-1-4-7 Diminished M7 chord	TSC4-18A

0-5-8-9 Third inversion major-minor tetrachord	0-3-8-11 Second inversion major-minor tetrachord	0-1-4-9 First inversion major-minor tetrachord	0-3-4-7 Major-minor tetrachord	TSC4-17

0-5-7-11 0-5-7-11 tetrachord	0-1-6-8 0-1-6-8 tetrachord	0-4-5-10 0-4-5-10 tetrachord	0-2-6-7 0-2-6-7 tetrachord	TSC4-16B

Names

Appendix A continues on the next page.


to 70%
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to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-4-7-8
37.22 %
0-4-8-11
38.06 %
0-1-5-9
36.67 %
0-3-4-8
34.44 %
0-4-5-8
34.44 %
0-4-8-9
36.67 %
0-3-7-11
38.06 %
0-1-4-8
37.22 %
0-5-8-11
32.74 %
0-1-6-9
34.13 %
0-3-4-9
35.79 %
0-3-6-7
36.35 %
TSC4-19B
TSC4-19A
TSC4-18B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-19B				TSC4-19A				TSC4-18B			
0-4-7-8	0-4-8-11	0-1-5-9	0-3-4-8	0-4-5-8	0-4-8-9	0-3-7-11	0-1-4-8	0-5-8-11	0-1-6-9	0-3-4-9	0-3-6-7
7 [150.00]	11 [150.00]	0 [150.00]	3 [150.00]	0 [183.33]	4 [183.33]	7 [183.33]	8 [183.33]	0 [183.33]	1 [183.33]	4 [183.33]	7 [183.33]
0 [133.33]	0 [133.33]	1 [133.33]	0 [133.33]	4 [133.33]	0 [133.33]	3 [133.33]	0 [133.33]	3 [103.33]	4 [103.33]	7 [103.33]	10 [103.33]
4 [133.33]	4 [133.33]	5 [133.33]	4 [133.33]	8 [133.33]	8 [133.33]	11 [133.33]	4 [133.33]	5 [100.00]	0 [100.00]	0 [100.00]	0 [100.00]
8 [133.33]	8 [133.33]	9 [133.33]	8 [133.33]	5 [100.00]	9 [100.00]	0 [100.00]	1 [100.00]	8 [100.00]	6 [100.00]	3 [100.00]	3 [100.00]
11 [83.33]	3 [83.33]	4 [83.33]	7 [83.33]	3 [70.00]	7 [70.00]	10 [70.00]	11 [70.00]	11 [100.00]	9 [100.00]	9 [100.00]	6 [100.00]
2 [82.50]	6 [82.50]	7 [82.50]	10 [82.50]	7 [62.50]	11 [62.50]	2 [62.50]	3 [62.50]	6 [70.00]	7 [70.00]	10 [70.00]	1 [70.00]
3 [50.00]	7 [50.00]	8 [50.00]	11 [50.00]	11 [50.00]	3 [50.00]	6 [50.00]	7 [50.00]	7 [62.50]	8 [62.50]	11 [62.50]	2 [62.50]
6 [32.50]	2 [32.50]	3 [32.50]	2 [32.50]	9 [33.33]	1 [33.33]	4 [33.33]	5 [33.33]	9 [53.33]	10 [53.33]	1 [53.33]	4 [53.33]
10 [32.50]	10 [32.50]	11 [32.50]	6 [32.50]	2 [32.50]	2 [32.50]	1 [32.50]	2 [32.50]	4 [33.33]	5 [33.33]	8 [33.33]	11 [33.33]
5 [20.00]	9 [20.00]	10 [20.00]	1 [20.00]	6 [32.50]	6 [32.50]	5 [32.50]	6 [32.50]	10 [32.50]	11 [32.50]	2 [32.50]	5 [32.50]
9 [12.50]	1 [12.50]	2 [12.50]	5 [12.50]	10 [32.50]	10 [32.50]	9 [32.50]	10 [32.50]	1 [12.50]	2 [12.50]	5 [12.50]	8 [12.50]
								2 [12.50]	3 [12.50]	6 [12.50]	9 [12.50]

to 220%												to 220%
to 210%												to 210%
to 200%												to 200%
to 190%												to 190%
to 180%												to 180%
to 170%												to 170%
to 160%												to 160%
to 150%												to 150%
to 140%												to 140%
to 130%												to 130%
to 120%												to 120%
to 110%												to 110%
to 100%												to 100%
to 90%												to 90%
to 80%	0-4-7-8 76.79 %	0-4-8-11 76.79 %	0-1-5-9 76.79 %	0-3-4-8 76.79 %								to 80%
to 70%												to 70%
to 60%												to 60%
to 50%												to 50%
to 40%												to 40%
to 30%												to 30%
to 20%					0-4-5-8 12.50 %	0-4-8-9 12.50 %	0-3-7-11 12.50 %	0-1-4-8 12.50 %	0-5-8-11 12.50 %	0-1-6-9 12.50 %	0-3-4-9 12.50 %	0-3-6-7 12.50 %	to 20%
to 10%													to 10%
0% / 2 equal roots													0% / 2 equal roots
0% / 3 equal roots													0% / 3 equal roots
0% / 4+ equal roots													0% / 4+ equal roots
Root clarity	TSC4-19B				TSC4-19A				TSC4-18B				Root clarity

TSC4-19B				TSC4-19A				TSC4-18B			
0-4-7-8	0-4-8-11	0-1-5-9	0-3-4-8	0-4-5-8	0-4-8-9	0-3-7-11	0-1-4-8	0-5-8-11	0-1-6-9	0-3-4-9	0-3-6-7
0 [275.00]	4 [275.00]	5 [275.00]	8 [275.00]	5 [175.00]	9 [175.00]	0 [175.00]	1 [175.00]	5 [175.00]	6 [175.00]	9 [175.00]	0 [175.00]
4 [155.56]	0 [155.56]	1 [155.56]	0 [155.56]	0 [155.56]	0 [155.56]	3 [155.56]	0 [155.56]	8 [155.56]	9 [155.56]	0 [155.56]	3 [155.56]
8 [155.56]	8 [155.56]	9 [155.56]	4 [155.56]	4 [155.56]	4 [155.56]	7 [155.56]	4 [155.56]	1 [155.00]	2 [155.00]	5 [155.00]	8 [155.00]
7 [100.00]	11 [100.00]	0 [100.00]	3 [100.00]	8 [155.56]	8 [155.56]	11 [155.56]	8 [155.56]	10 [123.75]	11 [123.75]	2 [123.75]	5 [123.75]
9 [81.67]	1 [81.67]	2 [81.67]	5 [81.67]	10 [123.75]	2 [123.75]	5 [123.75]	6 [123.75]	0 [100.00]	0 [100.00]	3 [100.00]	6 [100.00]
5 [72.92]	9 [72.92]	10 [72.92]	1 [72.92]	1 [97.22]	5 [97.22]	8 [97.22]	9 [97.22]	11 [100.00]	1 [100.00]	4 [100.00]	7 [100.00]
1 [50.00]	5 [50.00]	6 [50.00]	9 [50.00]	9 [50.00]	1 [50.00]	4 [50.00]	5 [50.00]	4 [97.22]	5 [97.22]	8 [97.22]	11 [97.22]
2 [37.92]	2 [37.92]	3 [37.92]	2 [37.92]	2 [37.92]	6 [37.92]	1 [37.92]	2 [37.92]	7 [62.22]	8 [62.22]	11 [62.22]	2 [62.22]
6 [37.92]	6 [37.92]	7 [37.92]	6 [37.92]	6 [37.92]	10 [37.92]	9 [37.92]	10 [37.92]	2 [20.00]	3 [20.00]	6 [20.00]	9 [20.00]
10 [37.92]	10 [37.92]	11 [37.92]	10 [37.92]	7 [20.00]	11 [20.00]	2 [20.00]	3 [20.00]	3 [12.50]	4 [12.50]	1 [12.50]	1 [12.50]
3 [33.33]	7 [33.33]	8 [33.33]	11 [33.33]	3 [12.50]	7 [12.50]	10 [12.50]	11 [12.50]	6 [12.50]	7 [12.50]	7 [12.50]	4 [12.50]
								9 [12.50]	10 [12.50]	10 [12.50]	10 [12.50]

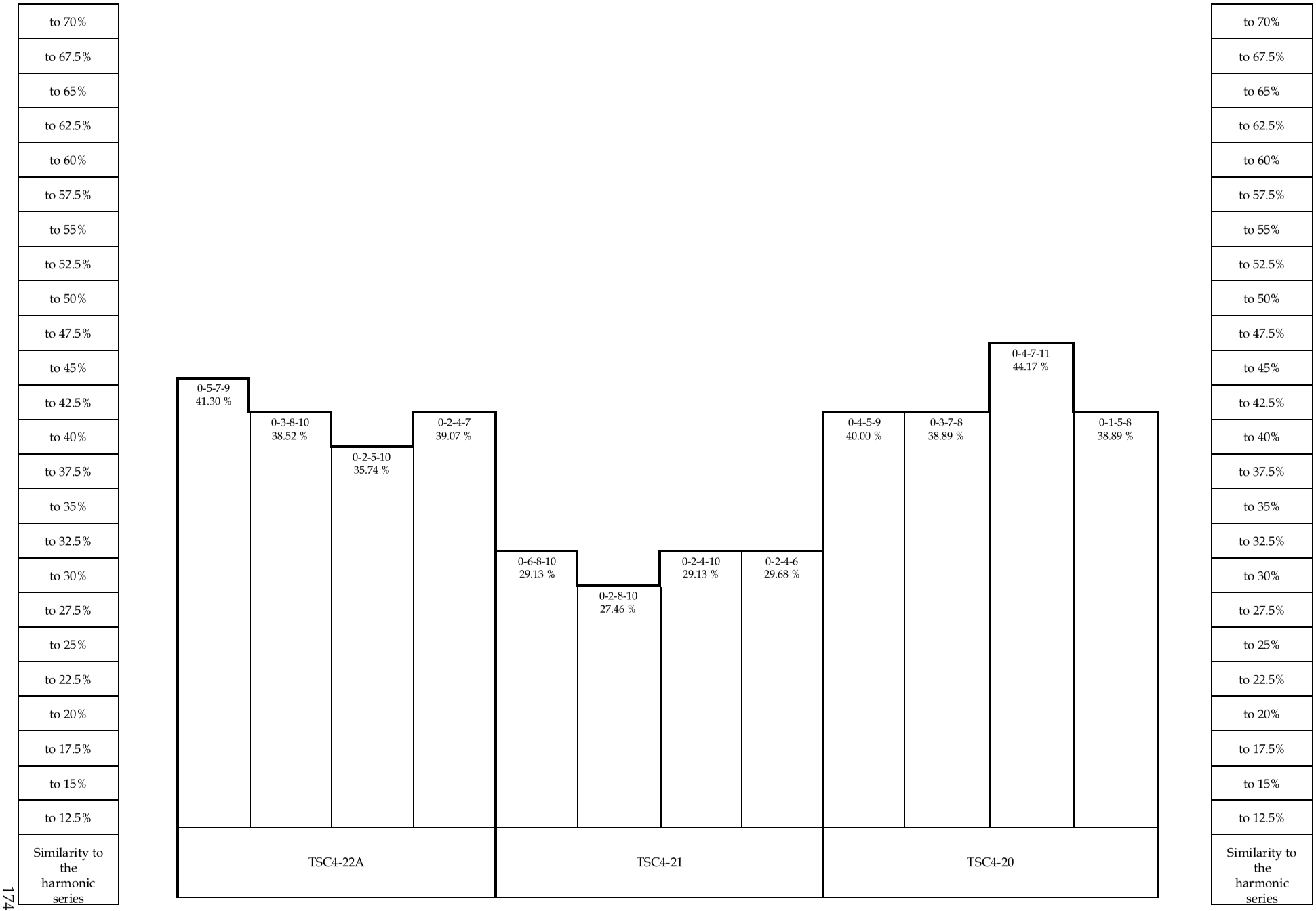
Root candidates and weights

Names

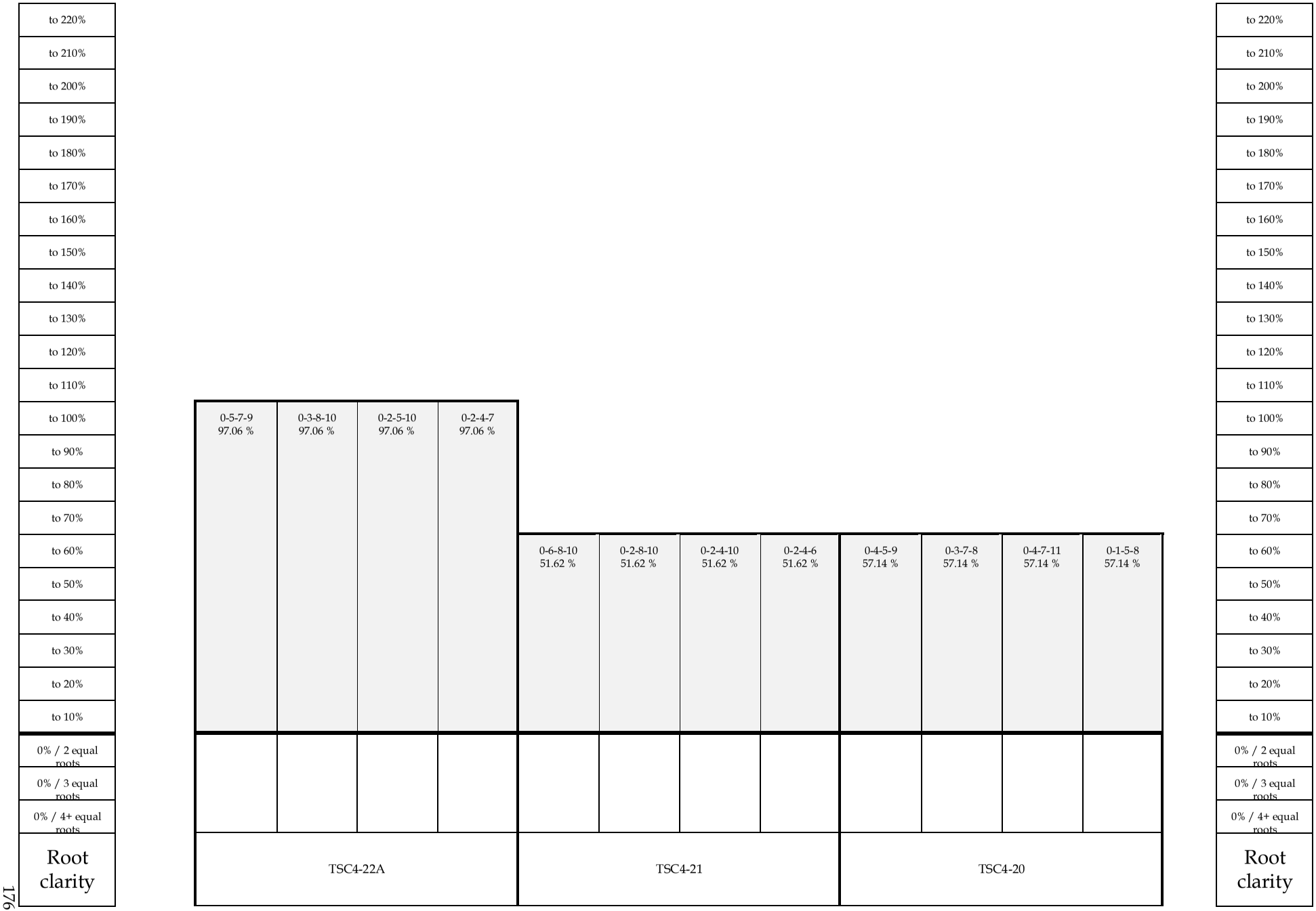
TSC4-19B					TSC4-19A					TSC4-18B				
0-4-7-8 0-4-7-8 tetrachord	0-4-8-11 Augmented major seventh	0-1-5-9 0-1-5-9 tetrachord	0-3-4-8 0-3-4-8 tetrachord		0-4-5-8 Second inversion minor-major seventh	0-4-8-9 First inversion minor-major seventh	0-3-7-11 Minor-major seventh	0-1-4-8 Third inversion minor-major seventh		0-5-8-11 0-5-8-11 tetrachord	0-1-6-9 0-1-6-9 tetrachord	0-3-4-9 0-3-4-9 tetrachord	0-3-6-7 0-3-6-7 tetrachord	

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-7-9
41.30 %
0-3-8-10
38.52 %
0-2-5-10
35.74 %
0-2-4-7
39.07 %
0-6-8-10
29.13 %
0-2-8-10
27.46 %
0-2-4-10
29.13 %
0-2-4-6
29.68 %
0-4-5-9
40.00 %
0-3-7-8
38.89 %
0-4-7-11
44.17 %
0-1-5-8
38.89 %
TSC4-22A
TSC4-21
TSC4-20
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-22A				TSC4-21				TSC4-20			
0-5-7-9	0-3-8-10	0-2-5-10	0-2-4-7	0-6-8-10	0-2-8-10	0-2-4-10	0-2-4-6	0-4-5-9	0-3-7-8	0-4-7-11	0-1-5-8
7 [182.50]	10 [182.50]	0 [182.50]	2 [182.50]	10 [165.83]	0 [165.83]	2 [165.83]	4 [165.83]	4 [183.33]	7 [183.33]	11 [183.33]	0 [183.33]
0 [150.00]	3 [150.00]	5 [150.00]	7 [150.00]	0 [145.83]	2 [145.83]	4 [145.83]	6 [145.83]	0 [150.00]	3 [150.00]	7 [150.00]	8 [150.00]
9 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	8 [132.50]	10 [132.50]	0 [132.50]	2 [132.50]	9 [133.33]	0 [133.33]	4 [133.33]	5 [133.33]
5 [120.00]	8 [120.00]	10 [120.00]	0 [120.00]	6 [120.00]	8 [120.00]	10 [120.00]	0 [120.00]	5 [100.00]	8 [100.00]	0 [100.00]	1 [100.00]
4 [83.33]	7 [83.33]	9 [83.33]	11 [83.33]	4 [53.33]	6 [53.33]	8 [53.33]	10 [53.33]	7 [82.50]	10 [82.50]	2 [82.50]	3 [82.50]
2 [62.50]	5 [62.50]	7 [62.50]	9 [62.50]	1 [50.00]	3 [50.00]	5 [50.00]	1 [50.00]	11 [62.50]	2 [62.50]	6 [62.50]	7 [62.50]
11 [45.83]	2 [45.83]	4 [45.83]	6 [45.83]	3 [50.00]	5 [50.00]	7 [50.00]	7 [50.00]	1 [33.33]	4 [33.33]	3 [33.33]	4 [33.33]
1 [33.33]	4 [33.33]	6 [33.33]	8 [33.33]	5 [50.00]	7 [50.00]	9 [50.00]	9 [50.00]	8 [33.33]	11 [33.33]	8 [33.33]	9 [33.33]
3 [20.00]	1 [20.00]	3 [20.00]	5 [20.00]	7 [50.00]	9 [50.00]	11 [50.00]	11 [50.00]	2 [32.50]	5 [32.50]	9 [32.50]	10 [32.50]
10 [20.00]	6 [20.00]	8 [20.00]	10 [20.00]	2 [45.83]	4 [45.83]	6 [45.83]	8 [45.83]	3 [20.00]	1 [20.00]	5 [20.00]	6 [20.00]
								10 [20.00]	6 [20.00]	10 [20.00]	11 [20.00]
								6 [12.50]	9 [12.50]	1 [12.50]	2 [12.50]


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-7-9 97.06 %
0-3-8-10 97.06 %
0-2-5-10 97.06 %
0-2-4-7 97.06 %
0-6-8-10 51.62 %
0-2-8-10 51.62 %
0-2-4-10 51.62 %
0-2-4-6 51.62 %
0-4-5-9 57.14 %
0-3-7-8 57.14 %
0-4-7-11 57.14 %
0-1-5-8 57.14 %
TSC4-22A
TSC4-21
TSC4-20
Root clarity

TSC4-22A					TSC4-21				TSC4-20				
0-5-7-9	0-3-8-10	0-2-5-10	0-2-4-7		0-6-8-10	0-2-8-10	0-2-4-10	0-2-4-6	0-4-5-9	0-3-7-8	0-4-7-11	0-1-5-8	
5 [391.67]	8 [391.67]	10 [391.67]	0 [391.67]		8 [331.67]	10 [331.67]	0 [331.67]	2 [331.67]	5 [275.00]	8 [275.00]	0 [275.00]	1 [275.00]	
7 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]		6 [218.75]	8 [218.75]	10 [218.75]	0 [218.75]	9 [175.00]	0 [175.00]	4 [175.00]	5 [175.00]	
0 [175.00]	3 [175.00]	5 [175.00]	7 [175.00]		10 [198.75]	0 [198.75]	2 [198.75]	4 [198.75]	0 [155.56]	3 [155.56]	7 [155.56]	8 [155.56]	
9 [140.00]	0 [140.00]	2 [140.00]	4 [140.00]		0 [140.00]	2 [140.00]	4 [140.00]	6 [140.00]	2 [123.75]	5 [123.75]	9 [123.75]	10 [123.75]	
2 [81.67]	5 [81.67]	7 [81.67]	9 [81.67]		2 [62.22]	4 [62.22]	6 [62.22]	8 [62.22]	4 [100.00]	7 [100.00]	11 [100.00]	0 [100.00]	
10 [72.92]	1 [72.92]	3 [72.92]	5 [72.92]		4 [53.47]	6 [53.47]	8 [53.47]	10 [53.47]	10 [72.92]	1 [72.92]	5 [72.92]	6 [72.92]	
3 [53.47]	6 [53.47]	8 [53.47]	10 [53.47]		1 [50.00]	1 [50.00]	3 [50.00]	5 [50.00]	7 [37.92]	10 [37.92]	2 [37.92]	3 [37.92]	
1 [33.33]	4 [33.33]	1 [33.33]	3 [33.33]		3 [50.00]	5 [50.00]	7 [50.00]	7 [50.00]	1 [33.33]	4 [33.33]	3 [33.33]	4 [33.33]	
8 [33.33]	11 [33.33]	6 [33.33]	8 [33.33]		5 [50.00]	7 [50.00]	9 [50.00]	9 [50.00]	8 [33.33]	11 [33.33]	8 [33.33]	9 [33.33]	
11 [20.00]	2 [20.00]	4 [20.00]	6 [20.00]		11 [50.00]	7 [50.00]	9 [50.00]	11 [50.00]	6 [20.00]	2 [20.00]	1 [20.00]	2 [20.00]	
									11 [20.00]	9 [20.00]	6 [20.00]	7 [20.00]	
									3 [12.50]	6 [12.50]	10 [12.50]	11 [12.50]	

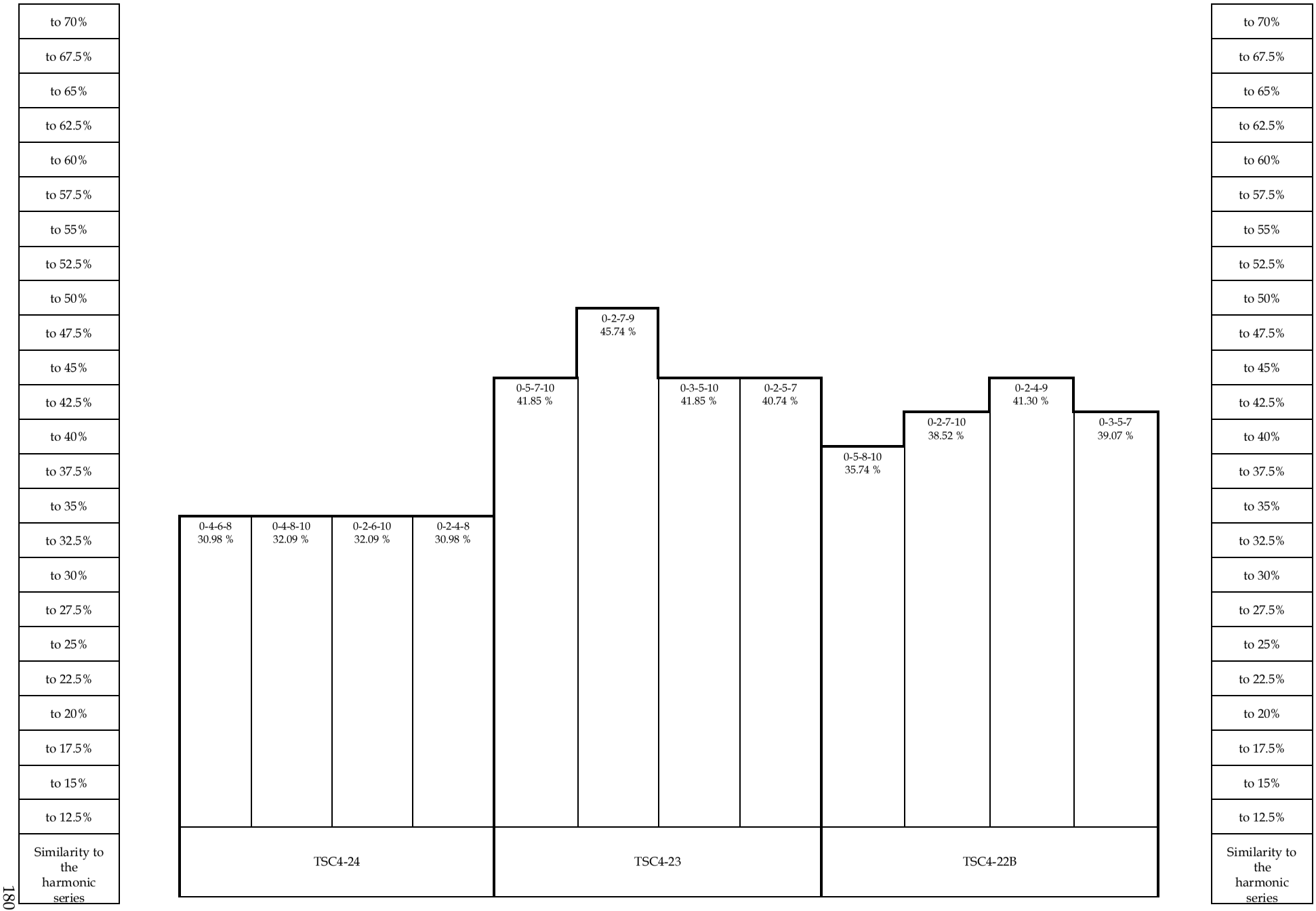
Root candidates and weights

Names

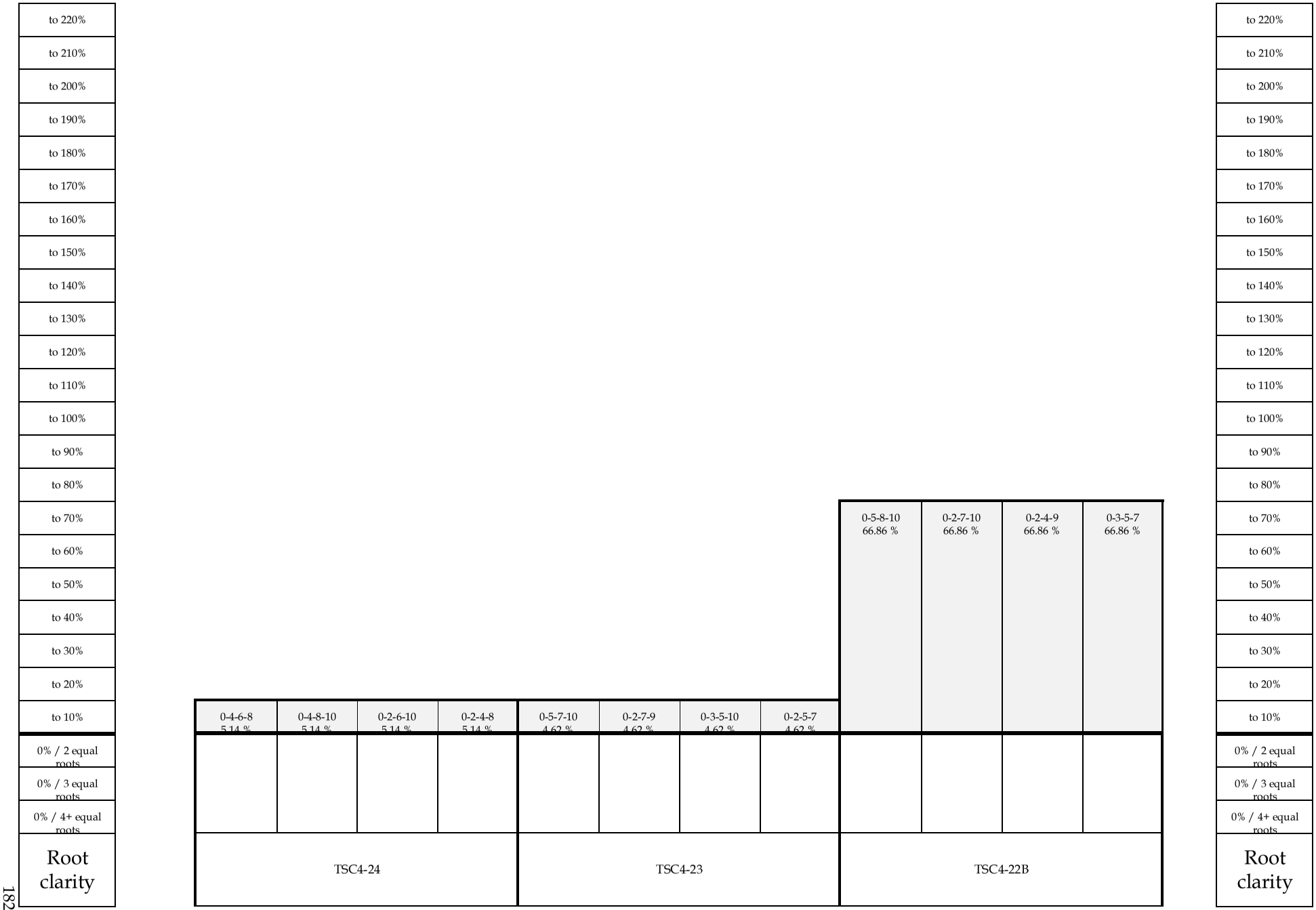
TSC4-22A					TSC4-21					TSC4-20				
0-5-7-9 Third inversion major add 2 tetrachord	0-3-8-10 Second inversion major add 2 tetrachord	0-2-5-10 First inversion major add 2 tetrachord	0-2-4-7 Major add 2 tetrachord Major add 9 tetrachord		0-6-8-10 0-6-8-10 tetrachord	0-2-8-10 0-2-8-10 tetrachord	0-2-4-10 0-2-4-10 tetrachord	0-2-4-6 0-2-4-6 tetrachord		0-4-5-9 Second inversion major seventh tetrachord	0-3-7-8 First inversion major seventh tetrachord	0-4-7-11 Major seventh tetrachord	0-1-5-8 Third inversion major seventh tetrachord	

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-4-6-8
30.98 %
0-4-8-10
32.09 %
0-2-6-10
32.09 %
0-2-4-8
30.98 %
0-5-7-10
41.85 %
0-2-7-9
45.74 %
0-3-5-10
41.85 %
0-2-5-7
40.74 %
0-5-8-10
35.74 %
0-2-7-10
38.52 %
0-2-4-9
41.30 %
0-3-5-7
39.07 %
TSC4-24
TSC4-23
TSC4-22B

TSC4-24				TSC4-23				TSC4-22B			
0-4-6-8	0-4-8-10	0-2-6-10	0-2-4-8	0-5-7-10	0-2-7-9	0-3-5-10	0-2-5-7	0-5-8-10	0-2-7-10	0-2-4-9	0-3-5-7
4 [153.33]	8 [153.33]	10 [153.33]	0 [153.33]	5 [170.00]	7 [170.00]	10 [170.00]	0 [170.00]	0 [195.83]	2 [195.83]	4 [195.83]	7 [195.83]
8 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	0 [162.50]	2 [162.50]	0 [162.50]	2 [162.50]	5 [150.00]	7 [150.00]	9 [150.00]	0 [150.00]
0 [133.33]	4 [133.33]	6 [133.33]	8 [133.33]	7 [162.50]	9 [162.50]	5 [162.50]	7 [162.50]	10 [132.50]	0 [132.50]	2 [132.50]	5 [132.50]
6 [132.50]	10 [132.50]	0 [132.50]	2 [132.50]	10 [120.00]	0 [120.00]	3 [120.00]	5 [120.00]	8 [120.00]	10 [120.00]	0 [120.00]	3 [120.00]
10 [65.83]	2 [65.83]	4 [65.83]	6 [65.83]	2 [95.83]	4 [95.83]	7 [95.83]	9 [95.83]	3 [70.00]	5 [70.00]	7 [70.00]	10 [70.00]
1 [50.00]	3 [50.00]	1 [50.00]	3 [50.00]	9 [45.83]	11 [45.83]	2 [45.83]	4 [45.83]	7 [62.50]	9 [62.50]	11 [62.50]	2 [62.50]
3 [50.00]	5 [50.00]	5 [50.00]	7 [50.00]	4 [33.33]	1 [33.33]	4 [33.33]	6 [33.33]	2 [45.83]	4 [45.83]	6 [45.83]	9 [45.83]
7 [50.00]	7 [50.00]	7 [50.00]	9 [50.00]	11 [33.33]	6 [33.33]	9 [33.33]	11 [33.33]	4 [33.33]	6 [33.33]	1 [33.33]	4 [33.33]
11 [50.00]	11 [50.00]	9 [50.00]	11 [50.00]	3 [20.00]	5 [20.00]	1 [20.00]	3 [20.00]	9 [33.33]	11 [33.33]	8 [33.33]	11 [33.33]
2 [32.50]	6 [32.50]	8 [32.50]	10 [32.50]	8 [20.00]	10 [20.00]	8 [20.00]	10 [20.00]	6 [20.00]	8 [20.00]	10 [20.00]	1 [20.00]


to 220%
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to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-4-6-8 5.14 %
0-4-8-10 5.14 %
0-2-6-10 5.14 %
0-2-4-8 5.14 %
0-5-7-10 4.62 %
0-2-7-9 4.62 %
0-3-5-10 4.62 %
0-2-5-7 4.62 %
0-5-8-10 66.86 %
0-2-7-10 66.86 %
0-2-4-9 66.86 %
0-3-5-7 66.86 %
TSC4-24
TSC4-23
TSC4-22B

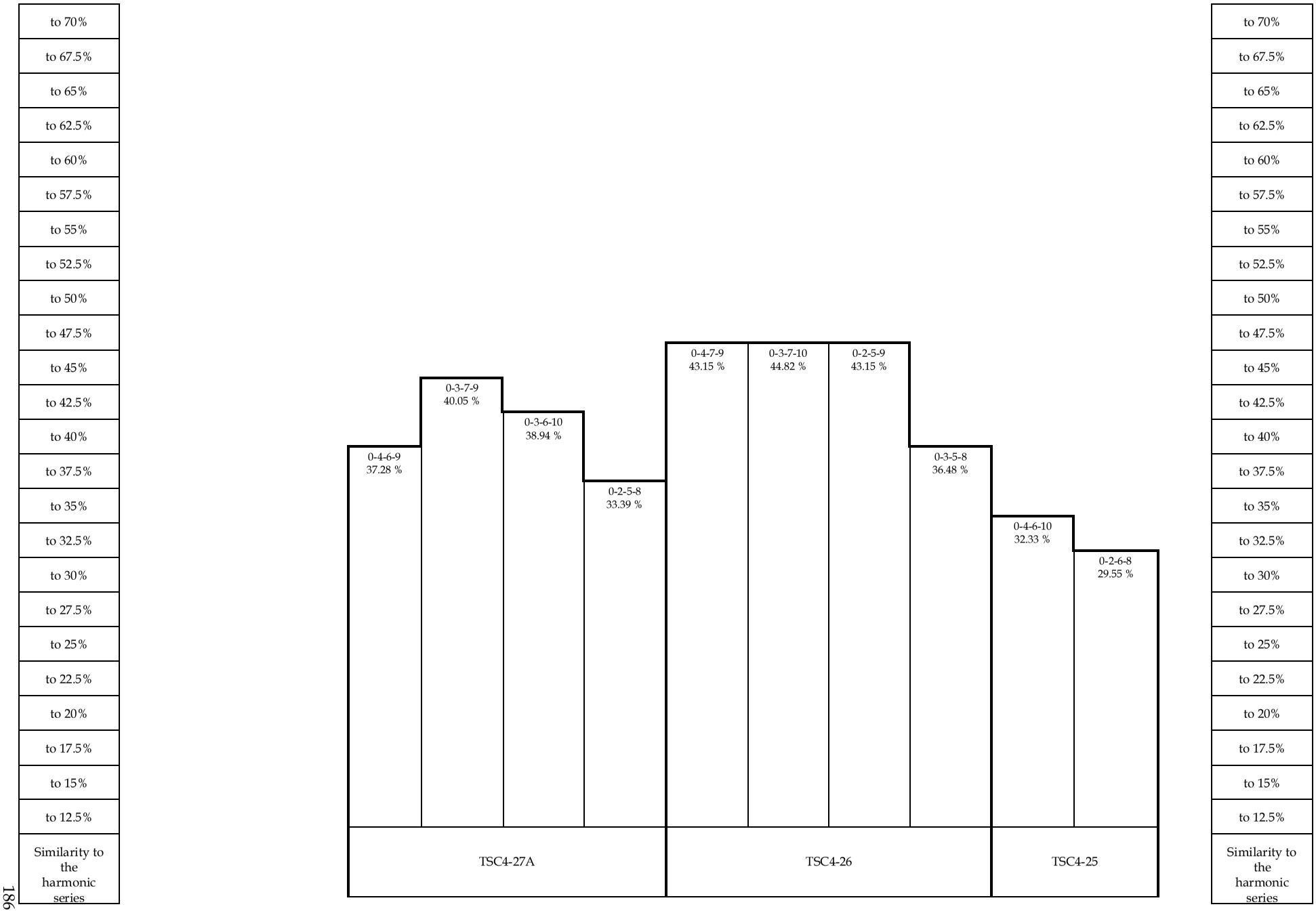
0-4-6-8	0-4-8-10	0-2-6-10	0-2-4-8	TSC4-24	0-5-7-10	0-2-7-9	0-3-5-10	0-2-5-7	TSC4-23	0-5-8-10	0-2-7-10	0-2-4-9	0-3-5-7	TSC4-22B
8 [230.00]	0 [230.00]	2 [230.00]	4 [230.00]		0 [255.00]	2 [255.00]	5 [255.00]	7 [255.00]		10 [365.00]	0 [365.00]	2 [365.00]	5 [365.00]	
4 [218.75]	8 [218.75]	10 [218.75]	0 [218.75]		5 [243.75]	0 [243.75]	3 [243.75]	0 [243.75]		8 [218.75]	10 [218.75]	0 [218.75]	3 [218.75]	
6 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]		10 [243.75]	7 [243.75]	10 [243.75]	5 [243.75]		5 [175.00]	7 [175.00]	9 [175.00]	0 [175.00]	
0 [155.56]	4 [155.56]	6 [155.56]	8 [155.56]		3 [143.75]	5 [143.75]	8 [143.75]	10 [143.75]		0 [140.00]	2 [140.00]	4 [140.00]	7 [140.00]	
2 [98.75]	6 [98.75]	8 [98.75]	10 [98.75]		7 [140.00]	9 [140.00]	0 [140.00]	2 [140.00]		1 [97.22]	3 [97.22]	5 [97.22]	8 [97.22]	
1 [50.00]	1 [50.00]	3 [50.00]	1 [50.00]		8 [53.47]	10 [53.47]	1 [53.47]	3 [53.47]		3 [72.92]	5 [72.92]	7 [72.92]	10 [72.92]	
5 [50.00]	3 [50.00]	5 [50.00]	5 [50.00]		1 [33.33]	3 [33.33]	6 [33.33]	1 [33.33]		6 [53.47]	8 [53.47]	10 [53.47]	1 [53.47]	
9 [50.00]	5 [50.00]	7 [50.00]	7 [50.00]		6 [33.33]	8 [33.33]	11 [33.33]	8 [33.33]		4 [33.33]	6 [33.33]	8 [33.33]	11 [33.33]	
11 [50.00]	9 [50.00]	11 [50.00]	9 [50.00]		2 [20.00]	4 [20.00]	2 [20.00]	4 [20.00]		2 [20.00]	4 [20.00]	6 [20.00]	2 [20.00]	
10 [37.92]	2 [37.92]	4 [37.92]	6 [37.92]		9 [20.00]	11 [20.00]	7 [20.00]	9 [20.00]		7 [20.00]	9 [20.00]	11 [20.00]	9 [20.00]	

Names

0-4-6-8	0-4-8-10	0-2-6-10	0-2-4-8	0-5-7-10	0-2-7-9	0-3-5-10	0-2-5-7	0-5-8-10	0-2-7-10	0-2-4-9	0-3-5-7
First inversion augmented seventh	Augmented seventh	Third inversion augmented seventh	Second inversion augmented seventh	Dominant seventh suspended fourth	Third inversion dominant seventh sus 4	Perfect quartal tetrachord	Genus Primum scale as chords	Mistuned Historical Church Bell 8 (calculated)	Mistuned Historical Church Bell 8 (aural)	Third inversion Mistuned Historical Church Bell 8 (aural)	Minor add 4 tetrachord
First inversion augmented dominant seventh	Augmented dominant seventh	Third inversion augmented dominant seventh	Second inversion augmented dominant seventh	Suspended seventh	Third inversion dominant seventh suspended fourth	Pentatonic seventh	First inversion dominant seventh sus 4				
First inversion augmented minor seventh	Augmented minor seventh	Third inversion augmented minor seventh	Second inversion augmented minor seventh	Dominant seventh sus 4	Third inversion suspended seventh	Second inversion dominant seventh suspended fourth	First inversion dominant seventh suspended fourth				
First inversion seventh augmented fifth	Seventh augmented fifth	Third inversion seventh augmented fifth	Second inversion seventh augmented fifth	Seventh suspended 4th		Second inversion suspended seventh	First inversion suspended seventh				
						Second inversion dominant seventh sus 4					
TSC4-24				TSC4-23				TSC4-22B			

Names

Appendix A continues on the next page.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to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-4-6-9
37.28 %
0-3-7-9
40.05 %
0-3-6-10
38.94 %
0-2-5-8
33.39 %
0-4-7-9
43.15 %
0-3-7-10
44.82 %
0-2-5-9
43.15 %
0-3-5-8
36.48 %
0-4-6-10
32.33 %
0-2-6-8
29.55 %
TSC4-27A
TSC4-26
TSC4-25
to 70%
to 67.5%
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to 62.5%
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to 20%
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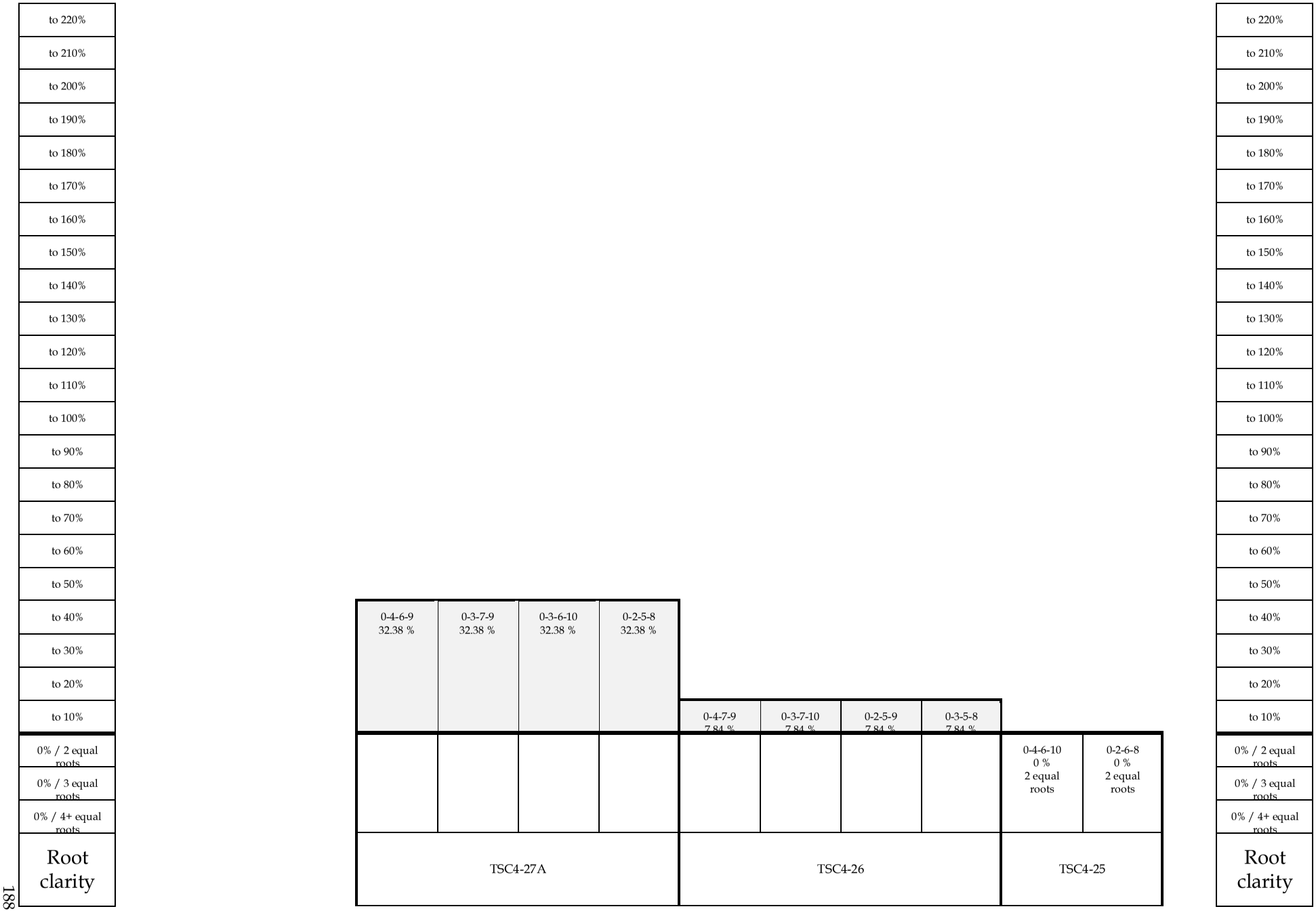
to 15%
to 12.5%
Similarity to the harmonic series

TSC4-27A				TSC4-26				TSC4-25	
0-4-6-9	0-3-7-9	0-3-6-10	0-2-5-8	0-4-7-9	0-3-7-10	0-2-5-9	0-3-5-8	0-4-6-10	0-2-6-8
4 [203.33]	7 [203.33]	10 [203.33]	0 [203.33]	4 [183.33]	7 [183.33]	9 [183.33]	0 [183.33]	4 [153.33]	0 [153.33]
6 [112.50]	9 [112.50]	0 [112.50]	2 [112.50]	7 [170.00]	10 [170.00]	0 [170.00]	3 [170.00]	10 [153.33]	6 [153.33]
0 [100.00]	0 [100.00]	3 [100.00]	5 [100.00]	9 [112.50]	0 [112.50]	2 [112.50]	5 [112.50]	0 [112.50]	2 [112.50]
9 [100.00]	3 [100.00]	6 [100.00]	8 [100.00]	0 [100.00]	3 [100.00]	5 [100.00]	8 [100.00]	6 [112.50]	8 [112.50]
1 [83.33]	4 [83.33]	7 [83.33]	9 [83.33]	11 [95.83]	2 [95.83]	4 [95.83]	7 [95.83]	2 [65.83]	4 [65.83]
7 [70.00]	10 [70.00]	1 [70.00]	3 [70.00]	2 [82.50]	5 [82.50]	7 [82.50]	10 [82.50]	8 [65.83]	10 [65.83]
11 [62.50]	2 62.50]	5 [62.50]	7 [62.50]	1 [33.33]	4 [33.33]	1 [33.33]	4 [33.33]	1 [50.00]	1 [50.00]
10 [53.33]	1 [53.33]	4 [53.33]	6 [53.33]	8 [33.33]	11 [33.33]	6 [33.33]	9 [33.33]	5 [50.00]	3 [50.00]
8 [45.83]	11 [45.83]	2 [45.83]	4 [45.83]	5 [20.00]	1 [20.00]	3 [20.00]	1 [20.00]	7 [50.00]	7 [50.00]
2 [32.50]	5 [32.50]	8 [32.50]	10 [32.50]	10 [20.00]	8 [20.00]	10 [20.00]	6 [20.00]	11 [50.00]	9 [50.00]
				6 [12.50]	9 [12.50]	11 [12.50]	2 [12.50]		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0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-4-6-9 32.38 %
0-3-7-9 32.38 %
0-3-6-10 32.38 %
0-2-5-8 32.38 %
0-4-7-9 7.84 %
0-3-7-10 7.84 %
0-2-5-9 7.84 %
0-3-5-8 7.84 %
0-4-6-10 0 % 2 equal roots
0-2-6-8 0 % 2 equal roots
TSC4-27A
TSC4-26
TSC4-25
Root clarity

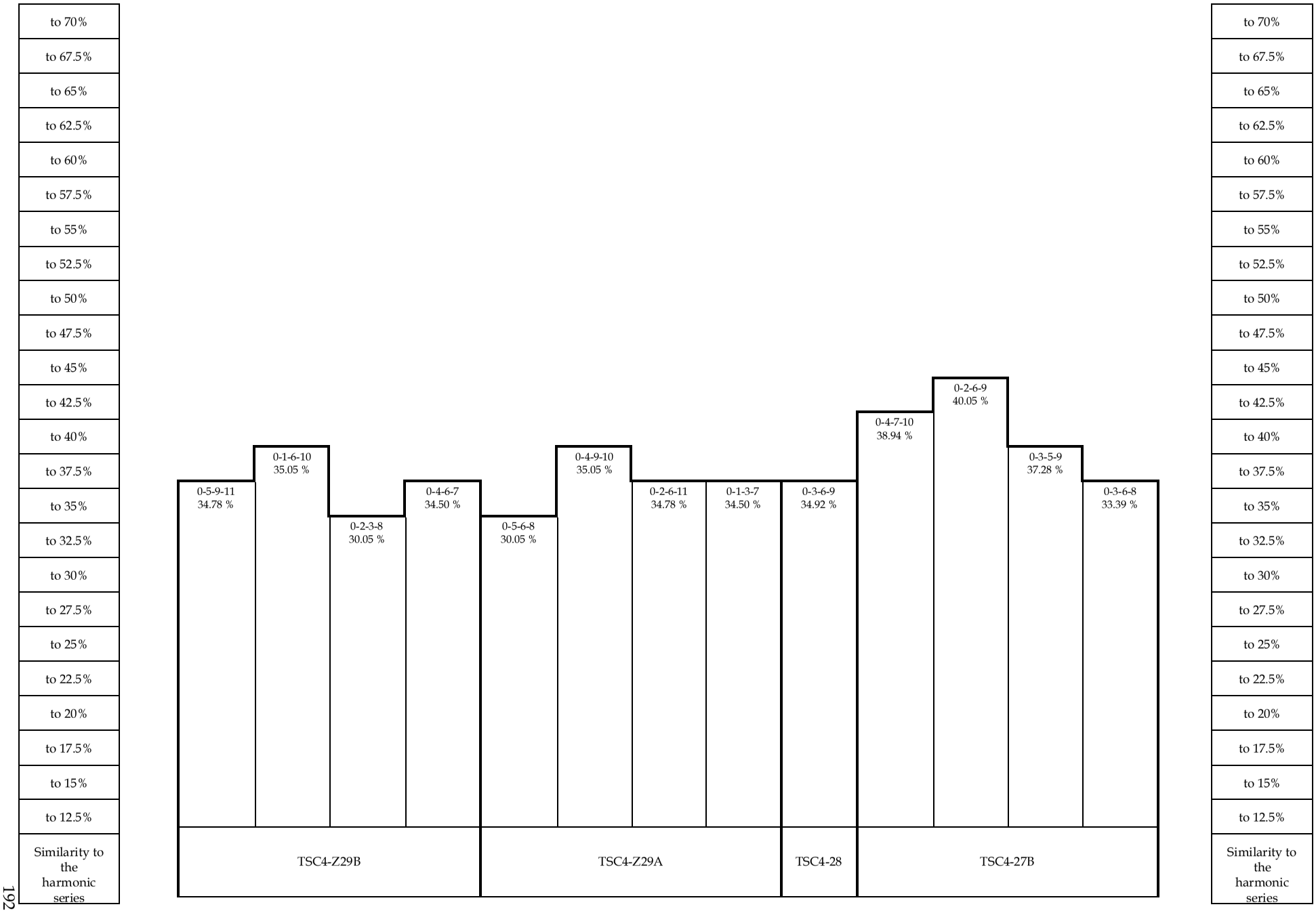
TSC4-27A				TSC4-26				TSC4-25	
0-4-6-9	0-3-7-9	0-3-6-10	0-2-5-8	0-4-7-9	0-3-7-10	0-2-5-9	0-3-5-8	0-4-6-10	0-2-6-8
2 [231.67]	5 [231.67]	8 [231.67]	10 [231.67]	0 [275.00]	3 [275.00]	5 [275.00]	8 [275.00]	0 [230.00]	2 [230.00]
9 [175.00]	0 [175.00]	3 [175.00]	5 [175.00]	9 [255.00]	0 [255.00]	2 [255.00]	5 [255.00]	6 [230.00]	8 [230.00]
0 [155.56]	3 [155.56]	6 [155.56]	8 [155.56]	5 [143.75]	8 [143.75]	10 [143.75]	1 [143.75]	4 [131.25]	0 [131.25]
6 [140.00]	9 [140.00]	0 [140.00]	2 [140.00]	7 [131.25]	10 [131.25]	0 [131.25]	3 [131.25]	10 [131.25]	6 [131.25]
4 [131.25]	7 [131.25]	10 [131.25]	0 [131.25]	2 [123.75]	5 [123.75]	7 [123.75]	10 [123.75]	2 [98.75]	4 [98.75]
5 [97.22]	8 [97.22]	11 [97.22]	1 [97.22]	4 [100.00]	7 [100.00]	9 [100.00]	0 [100.00]	8 [98.75]	10 [98.75]
11 [81.67]	2 [81.67]	5 [81.67]	7 [81.67]	3 [33.33]	6 [33.33]	1 [33.33]	4 [33.33]	3 [50.00]	1 [50.00]
8 [62.22]	11 [62.22]	2 [62.22]	4 [62.22]	8 [33.33]	11 [33.33]	8 [33.33]	11 [33.33]	5 [50.00]	5 [50.00]
7 [12.50]	1 [12.50]	1 [12.50]	3 [12.50]	6 [20.00]	2 [20.00]	4 [20.00]	2 [20.00]	9 [50.00]	7 [50.00]
10 [12.50]	10 [12.50]	4 [12.50]	6 [12.50]	11 [20.00]	9 [20.00]	11 [20.00]	7 [20.00]	11 [50.00]	11 [50.00]
				10 [12.50]	1 [12.50]	3 [12.50]	6 [12.50]		

Names

0-4-6-9	0-3-7-9	0-3-6-10	0-2-5-8	0-4-7-9	0-3-7-10	0-2-5-9	0-3-5-8	0-4-6-10	0-2-6-8
0-4-6-9 tetrachord	Minor sixth tetrachord	Half-diminished seventh " Tristan" chord	0-2-5-8 tetrachord	Major triad add six Major sixth tetrachord Sixth	Minor seventh tetrachord Minor seventh chord Minor subdominant seventh Minor-minor seventh Third inversion major triad add six	Second inversion major triad add six	First inversion major triad add six Golden Section scale as chords	French sixth Dominant seventh plus four Messiaen's truncated mode 6 Inverse scale	First inversion French sixth First inversion dominant seventh plus four Messiaen's truncated mode 6 scale
TSC4-27A				TSC4-26				TSC4-25	

Names

Appendix A continues on the next page.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to 22.5%
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to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-5-9-11
34.78 %
0-1-6-10
35.05 %
0-2-3-8
30.05 %
0-4-6-7
34.50 %
0-5-6-8
30.05 %
0-4-9-10
35.05 %
0-2-6-11
34.78 %
0-1-3-7
34.50 %
0-3-6-9
34.92 %
0-4-7-10
38.94 %
0-2-6-9
40.05 %
0-3-5-9
37.28 %
0-3-6-8
33.39 %
TSC4-Z29B
TSC4-Z29A
TSC4-28
TSC4-27B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
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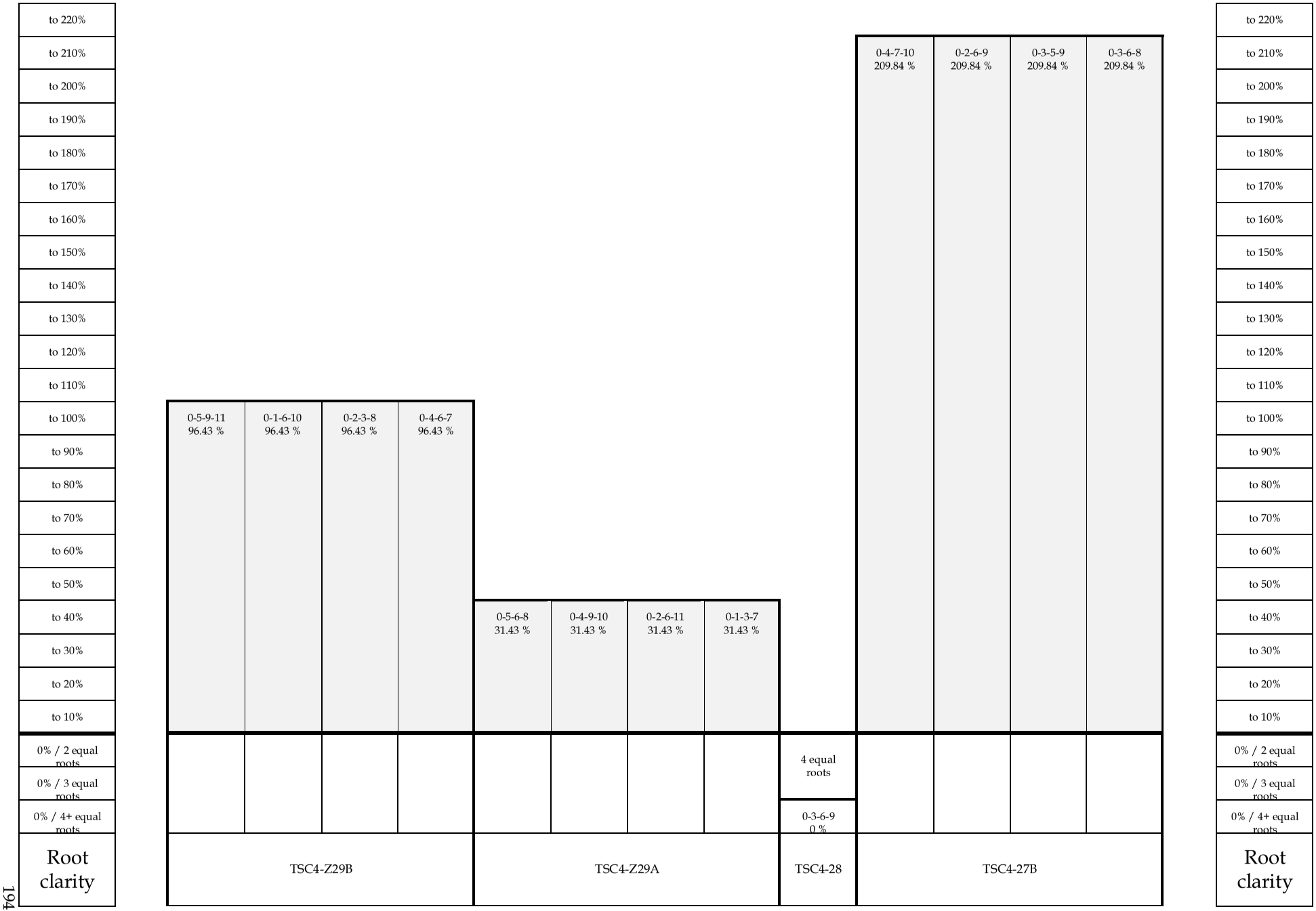
Similarity to the harmonic series

TSC4-Z29B				TSC4-Z29A			TSC4-28	TSC4-27B		
0-5-9-11	0-1-6-10	0-2-3-8	0-4-6-7	0-5-6-8	0-4-9-10	0-2-6-11	0-1-3-7	0-3-6-9	0-4-7-10	0-2-6-9	0-3-5-9	0-3-6-8
9 [153.33]	10 [153.33]	0 [153.33]	4 [153.33]	0 [183.33]	4 [183.33]	6 [183.33]	7 [183.33]	1 [103.33]	7 [150.00]	9 [150.00]	0 [150.00]	3 [150.00]
0 [150.00]	1 [150.00]	3 [150.00]	7 [150.00]	6 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]	4 [103.33]	4 [133.33]	6 [133.33]	9 [133.33]	0 [133.33]
11 [112.50]	0 [112.50]	2 [112.50]	6 [112.50]	8 [112.50]	0 [112.50]	2 [112.50]	3 [112.50]	7 [103.33]	10 [120.00]	0 [120.00]	3 [120.00]	6 [120.00]
5 [100.00]	6 [100.00]	8 [100.00]	0 [100.00]	5 [100.00]	9 [100.00]	11 [100.00]	0 [100.00]	10 [103.33]	2 [115.83]	4 [115.83]	7 [115.83]	10 [115.83]
4 [83.33]	5 [83.33]	7 [83.33]	11 [83.33]	3 [70.00]	7 [70.00]	9 [70.00]	10 [70.00]	0 [100.00]	0 [112.50]	2 [112.50]	5 [112.50]	8 [112.50]
7 [82.50]	8 [82.50]	10 [82.50]	2 [82.50]	10 [65.83]	2 [65.83]	4 [65.83]	5 [65.83]	3 [100.00]	11 [83.33]	1 [83.33]	4 [83.33]	7 [83.33]
3 [53.33]	4 [53.33]	6 [53.33]	10 [53.33]	7 [62.50]	11 [62.50]	1 [62.50]	2 [62.50]	6 [100.00]	5 [70.00]	7 [70.00]	10 [70.00]	1 [70.00]
6 [50.00]	7 [50.00]	9 [50.00]	1 [50.00]	4 [53.33]	8 [53.33]	10 [53.33]	11 [53.33]	9 [100.00]	8 [53.33]	10 [53.33]	1 [53.33]	4 [53.33]
1 [45.83]	2 [45.83]	4 [45.83]	8 [45.83]	1 [50.00]	5 [50.00]	7 [50.00]	8 [50.00]	2 [12.50]	6 [12.50]	8 [12.50]	2 [12.50]	2 [12.50]
10 [20.00]	11 [20.00]	1 [20.00]	5 [20.00]	9 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	5 [12.50]	9 [12.50]	11 [12.50]	11 [12.50]	5 [12.50]
2 [12.50]	3 [12.50]	5 [12.50]	9 [12.50]	2 [12.50]	6 [12.50]	8 [12.50]	9 [12.50]	8 [12.50]				
								11 [12.50]				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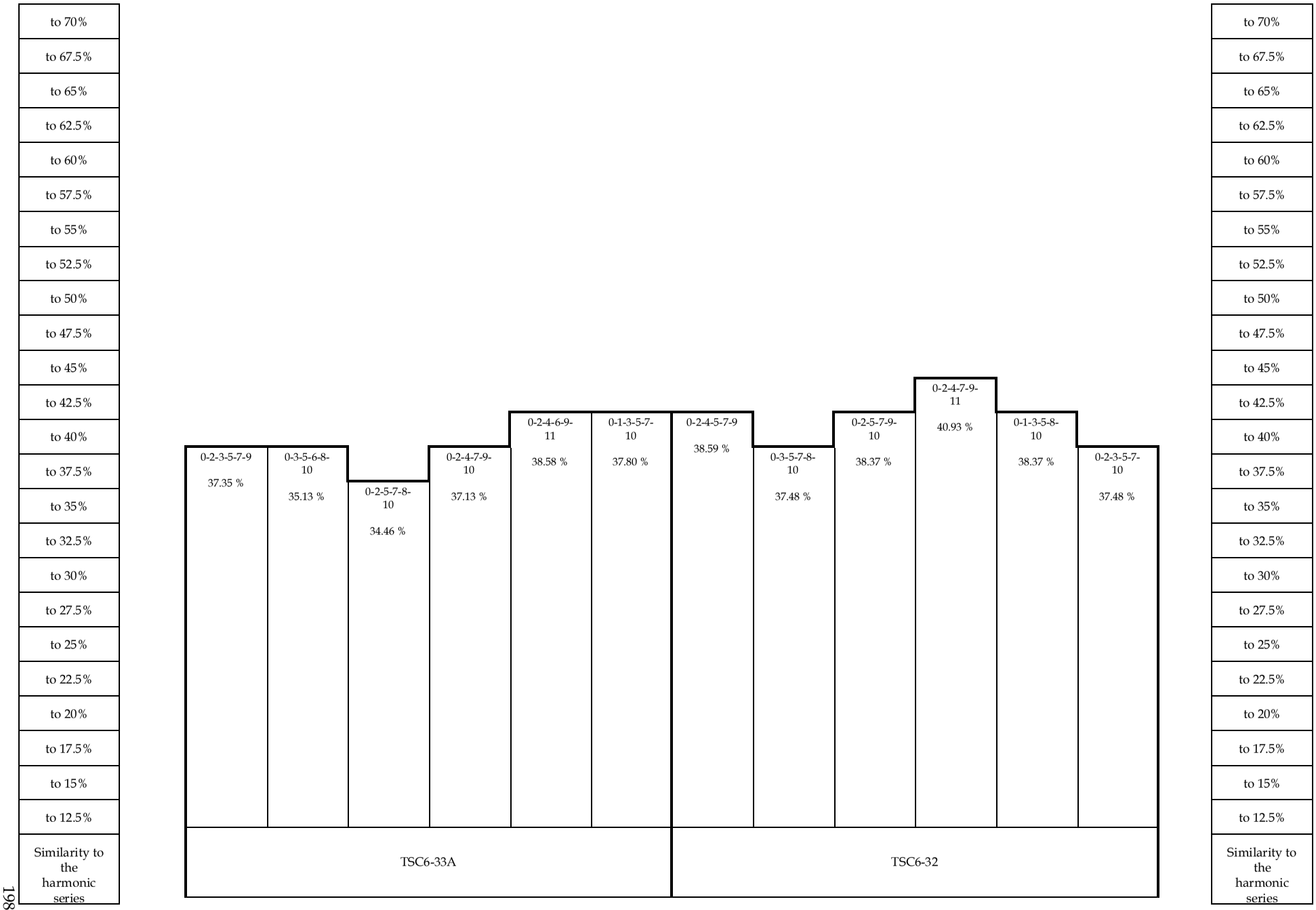
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-5-9-11 96.43 %
0-1-6-10 96.43 %
0-2-3-8 96.43 %
0-4-6-7 96.43 %
0-5-6-8 31.43 %
0-4-9-10 31.43 %
0-2-6-11 31.43 %
0-1-3-7 31.43 %
4 equal roots
0-3-6-9 0 %
0-4-7-10 209.84 %
0-2-6-9 209.84 %
0-3-5-9 209.84 %
0-3-6-8 209.84 %
TSC4-Z29B
TSC4-Z29A
TSC4-28
TSC4-27B
Root clarity

TSC4-Z29B				TSC4-Z29A			TSC4-28	TSC4-27B		
0-5-9-11	0-1-6-10	0-2-3-8	0-4-6-7	0-5-6-8	0-4-9-10	0-2-6-11	0-1-3-7	0-3-6-9	0-4-7-10	0-2-6-9	0-3-5-9	0-3-6-8
5 [275.00] 11 [140.00] 9 [131.25] 0 [100.00] 7 [98.75] 2 [81.67] 10 [72.92] 1 [62.22] 4 [50.00] 8 [33.33] 3 [12.50]	6 [275.00] 0 [140.00] 10 [131.25] 1 [100.00] 8 [98.75] 3 [81.67] 11 [72.92] 2 [62.22] 5 [50.00] 9 [33.33] 4 [12.50]	8 [275.00] 2 [140.00] 0 [131.25] 3 [100.00] 10 [98.75] 5 [81.67] 1 [72.92] 4 [62.22] 7 [50.00] 11 [33.33] 6 [12.50]	0 [275.00] 6 [140.00] 4 [131.25] 7 [100.00] 2 [98.75] 9 [81.67] 5 [72.92] 8 [62.22] 11 [50.00] 3 [33.33] 10 [12.50]	8 [230.00] 5 [175.00] 6 [131.25] 10 [123.75] 0 [100.00] 1 [97.22] 2 [62.22] 4 [53.47] 11 [50.00] 7 [20.00] 3 [12.50]	0 [230.00] 9 [175.00] 10 [131.25] 2 [123.75] 4 [100.00] 5 [97.22] 6 [62.22] 8 [53.47] 3 [50.00] 11 [20.00] 7 [12.50]	2 [230.00] 11 [175.00] 0 [131.25] 4 [123.75] 6 [100.00] 7 [97.22] 8 [62.22] 10 [53.47] 5 [50.00] 1 [20.00] 9 [12.50]	3 [230.00] 0 [175.00] 1 [131.25] 5 [123.75] 7 [100.00] 8 [97.22] 9 [62.22] 11 [53.47] 6 [50.00] 2 [20.00] 10 [12.50]	2 [155.00] 5 [155.00] 8 [155.00] 11 [155.00] 0 [100.00] 3 [100.00] 6 [100.00] 9 [100.00] 1 [12.50] 4 [12.50] 7 [12.50] 10 [12.50]	0 [406.67] 10 [131.25] 4 [100.00] 7 [100.00] 3 [97.22] 9 [81.67] 5 [72.92] 6 [62.22] 8 [53.47] 2 [37.92]	2 [406.67] 0 [131.25] 6 [100.00] 9 [100.00] 5 [97.22] 11 [81.67] 7 [72.92] 8 [62.22] 10 [53.47] 4 [37.92]	5 [406.67] 3 [131.25] 0 [100.00] 9 [100.00] 8 [97.22] 2 [81.67] 10 [72.92] 11 [62.22] 1 [53.47] 7 [37.92]	8 [406.67] 6 [131.25] 0 [100.00] 3 [100.00] 11 [97.22] 5 [81.67] 1 [72.92] 2 [62.22] 4 [53.47] 10 [37.92]

0-5-9-11 0-5-9-11 tetrachord	0-1-6-10 0-1-6-10 tetrachord	0-2-3-8 0-2-3-8 tetrachord	0-4-6-7 0-4-6-7 tetrachord	0-5-6-8 0-5-6-8 tetrachord	0-4-9-10 0-4-9-10 tetrachord	0-2-6-11 Raga Sumukam scale as chords	0-1-3-7 All-interval tetrachord B	0-3-6-9 Diminished seventh Fully-diminished seventh	0-4-7-10 Dominant seventh Major-minor seventh (in minor mode) Major-minor seventh chord Major-minor subdominant seventh German sixth	0-2-6-9 Third inversion dominant seventh chord Third inversion major-minor seventh (in minor mode) Third inversion German sixth	0-3-5-9 Second inversion dominant seventh chord Second inversion major-minor seventh (in minor mode) Second inversion German sixth	0-3-6-8 First inversion dominant seventh chord First inversion major-minor seventh (in minor mode) First inversion German sixth
TSC4-Z29B				TSC4-Z29A				TSC4-28	TSC4-27B			

Selected six-pitch chords (eleventh chords and hexachords)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to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-2-3-5-7-9
37.35 %
0-3-5-6-8-10
35.13 %
0-2-5-7-8-10
34.46 %
0-2-4-7-9-10
37.13 %
0-2-4-6-9-11
38.58 %
0-1-3-5-7-10
37.80 %
0-2-4-5-7-9
38.59 %
0-3-5-7-8-10
37.48 %
0-2-5-7-9-10
38.37 %
0-2-4-7-9-11
40.93 %
0-1-3-5-8-10
38.37 %
0-2-3-5-7-10
37.48 %
TSC6-33A
TSC6-32
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to 22.5%
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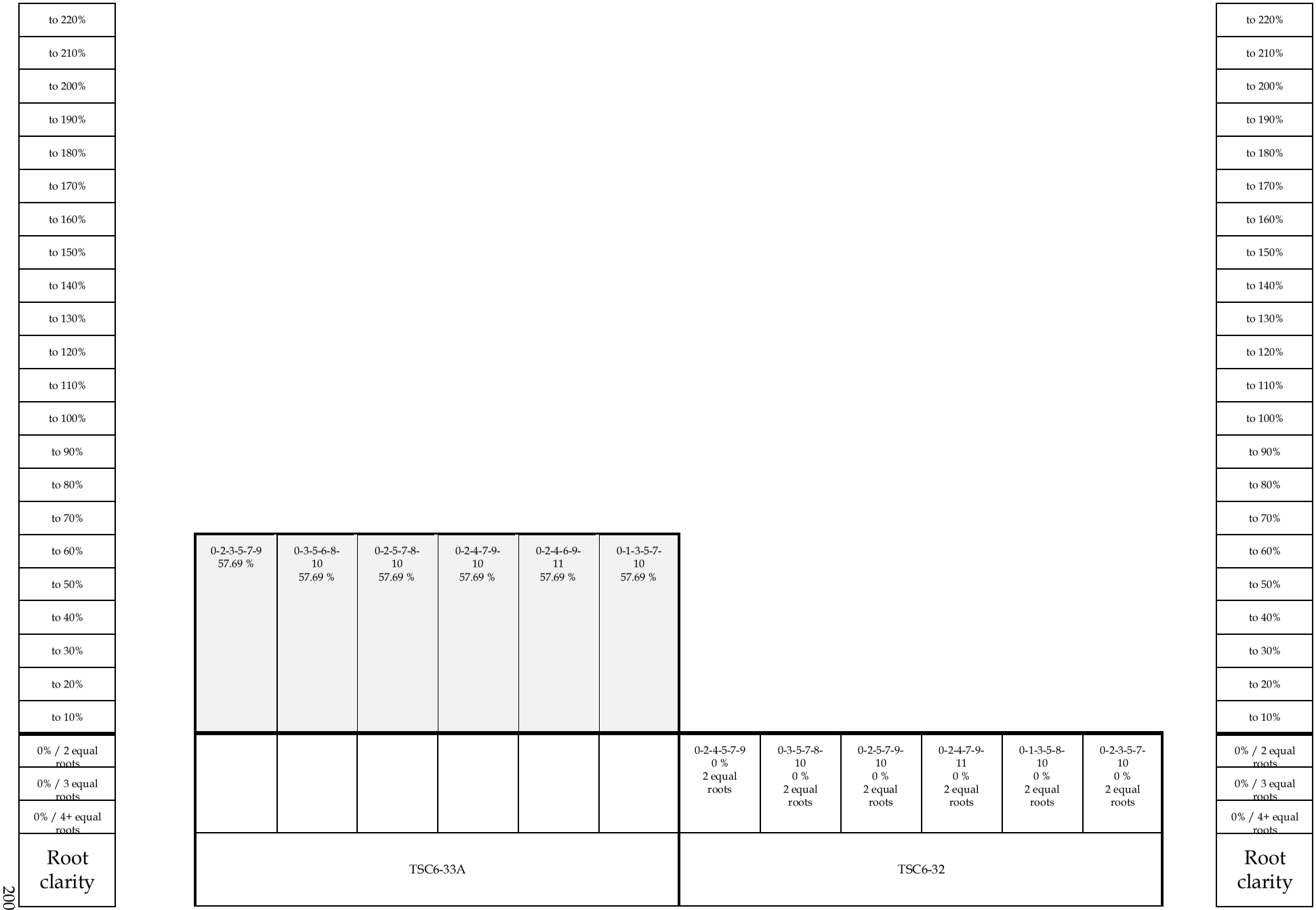
to 15%
to 12.5%
Similarity to the harmonic series

0-2-3-5-7-9	0-3-5-6-8-10	0-2-5-7-8-10	0-2-4-7-9-10	0-2-4-6-9-11	0-1-3-5-7-10	0-2-4-5-7-9	0-3-5-7-8-10	0-2-5-7-9-10	0-2-4-7-9-11	0-1-3-5-8-10	0-2-3-5-7-10
7 [215.83]	10 [215.83]	0 [215.83]	2 [215.83]	4 [215.83]	5 [215.83]	4 [195.83]	0 [195.83]	2 [195.83]	4 [195.83]	0 [195.83]	2 [195.83]
9 [195.83]	0 [195.83]	2 [195.83]	4 [195.83]	6 [195.83]	7 [195.83]	9 [195.83]	7 [195.83]	9 [195.83]	11 [195.83]	5 [195.83]	7 [195.83]
0 [170.00]	3 [170.00]	5 [170.00]	7 [170.00]	9 [170.00]	10 [170.00]	2 [182.50]	5 [182.50]	0 [182.50]	2 [182.50]	3 [182.50]	0 [182.50]
2 [162.50]	5 [162.50]	7 [162.50]	9 [162.50]	11 [162.50]	0 [162.50]	7 [182.50]	10 [182.50]	7 [182.50]	9 [182.50]	10 [182.50]	5 [182.50]
5 [132.50]	8 [132.50]	10 [132.50]	0 [132.50]	2 [132.50]	3 [132.50]	0 [170.00]	3 [170.00]	5 [170.00]	7 [170.00]	8 [170.00]	10 [170.00]
3 [120.00]	6 [120.00]	8 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]	5 [120.00]	8 [120.00]	10 [120.00]	0 [120.00]	1 [120.00]	3 [120.00]
4 [95.83]	7 [95.83]	9 [95.83]	11 [95.83]	1 [95.83]	2 [95.83]	11 [95.83]	2 [95.83]	4 [95.83]	6 [95.83]	7 [95.83]	9 [95.83]
10 [70.00]	1 [70.00]	3 [70.00]	5 [70.00]	7 [70.00]	8 [70.00]	6 [45.83]	9 [45.83]	11 [45.83]	1 [45.83]	2 [45.83]	4 [45.83]
1 [53.33]	4 [53.33]	6 [53.33]	8 [53.33]	10 [53.33]	11 [53.33]	1 [33.33]	4 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	6 [33.33]
11 [45.83]	2 [45.83]	4 [45.83]	6 [45.83]	8 [45.83]	9 [45.83]	8 [33.33]	11 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]
6 [33.33]	9 [33.33]	11 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	3 [20.00]	1 [20.00]	3 [20.00]	5 [20.00]	6 [20.00]	1 [20.00]
						10 [20.00]	6 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	8 [20.00]
TSC6-33A						TSC6-32					


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-2-3-5-7-9 57.69 %
0-3-5-6-8-10 57.69 %
0-2-5-7-8-10 57.69 %
0-2-4-7-9-10 57.69 %
0-2-4-6-9-11 57.69 %
0-1-3-5-7-10 57.69 %
0-2-4-5-7-9 0 % 2 equal roots
0-3-5-7-8-10 0 % 2 equal roots
0-2-5-7-9-10 0 % 2 equal roots
0-2-4-7-9-11 0 % 2 equal roots
0-1-3-5-8-10 0 % 2 equal roots
0-2-3-5-7-10 0 % 2 equal roots
TSC6-33A
TSC6-32
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity

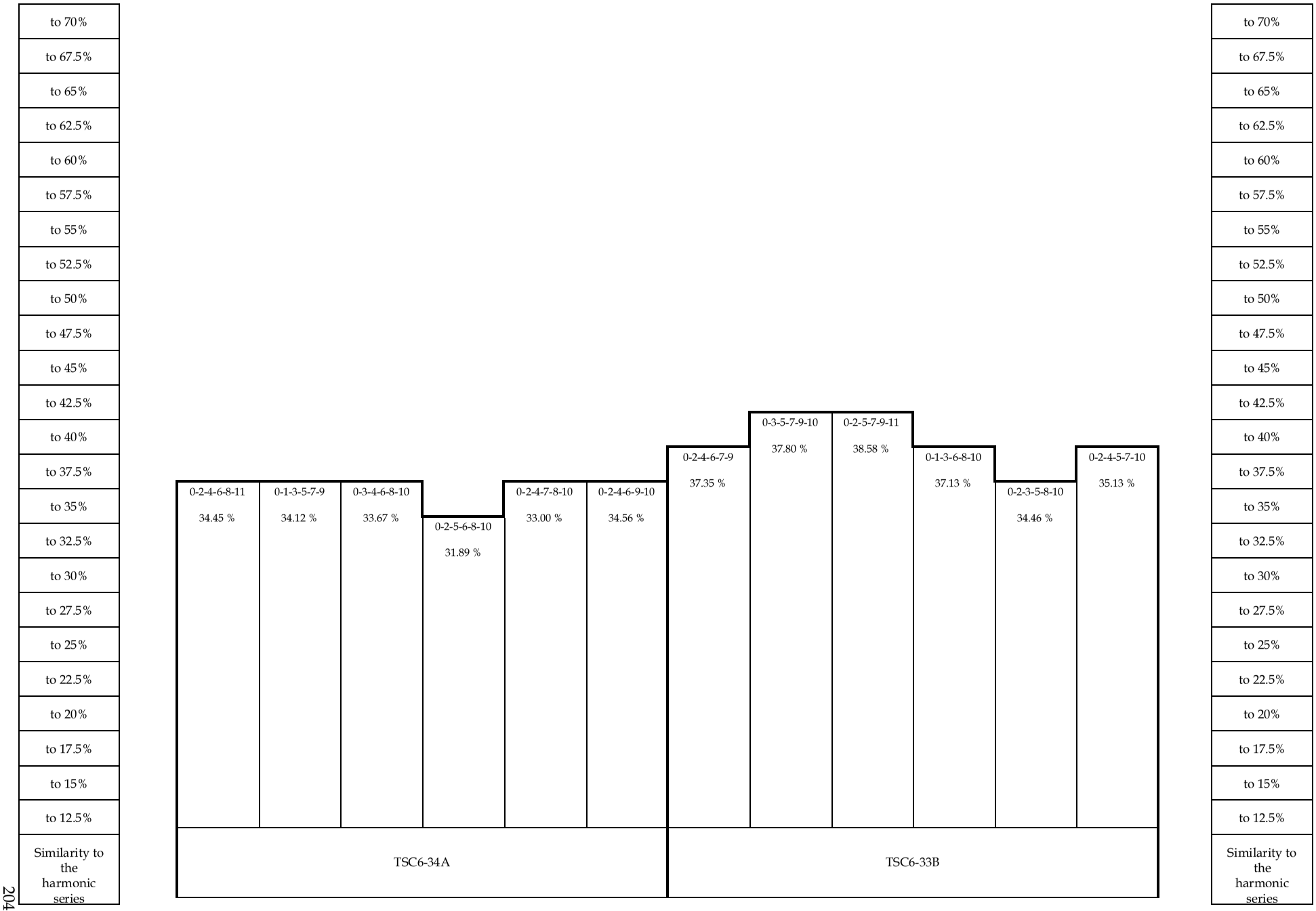
0-2-3-5-7-9	0-3-5-6-8-10	0-2-5-7-8-10	0-2-4-7-9-10	0-2-4-6-9-11	0-1-3-5-7-10	0-2-4-5-7-9	0-3-5-7-8-10	0-2-5-7-9-10	0-2-4-7-9-11	0-1-3-5-8-10	0-2-3-5-7-10
5 [575.56]	8 [575.56]	10 [575.56]	0 [575.56]	2 [575.56]	3 [575.56]	0 [391.67]	3 [391.67]	5 [391.67]	0 [391.67]	1 [391.67]	3 [391.67]
7 [365.00]	10 [365.00]	0 [365.00]	2 [365.00]	4 [365.00]	5 [365.00]	5 [391.67]	8 [391.67]	10 [391.67]	7 [391.67]	8 [391.67]	10 [391.67]
2 [255.00]	5 [255.00]	7 [255.00]	9 [255.00]	11 [255.00]	0 [255.00]	2 [365.00]	5 [365.00]	0 [365.00]	2 [365.00]	3 [365.00]	0 [365.00]
0 [243.75]	3 [243.75]	5 [243.75]	7 [243.75]	9 [243.75]	10 [243.75]	7 [365.00]	10 [365.00]	7 [365.00]	9 [365.00]	10 [365.00]	5 [365.00]
3 [218.75]	6 [218.75]	8 [218.75]	10 [218.75]	0 [218.75]	1 [218.75]	9 [255.00]	0 [255.00]	2 [255.00]	4 [255.00]	5 [255.00]	7 [255.00]
10 [143.75]	1 [143.75]	3 [143.75]	5 [143.75]	7 [143.75]	8 [143.75]	10 [143.75]	1 [143.75]	3 [143.75]	5 [143.75]	6 [143.75]	8 [143.75]
9 [140.00]	0 [140.00]	2 [140.00]	4 [140.00]	6 [140.00]	7 [140.00]	4 [140.00]	7 [140.00]	9 [140.00]	11 [140.00]	0 [140.00]	2 [140.00]
8 [97.22]	11 [97.22]	1 [97.22]	3 [97.22]	5 [97.22]	6 [97.22]	3 [53.47]	6 [53.47]	8 [53.47]	10 [53.47]	11 [53.47]	1 [53.47]
11 [62.22]	2 [62.22]	4 [62.22]	6 [62.22]	8 [62.22]	9 [62.22]	1 [33.33]	4 [33.33]	1 [33.33]	3 [33.33]	4 [33.33]	6 [33.33]
1 [53.47]	4 [53.47]	6 [53.47]	8 [53.47]	10 [53.47]	11 [53.47]	8 [33.33]	11 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]
4 [20.00]	7 [20.00]	9 [20.00]	11 [20.00]	1 [20.00]	2 [20.00]	6 [20.00]	2 [20.00]	4 [20.00]	1 [20.00]	2 [20.00]	4 [20.00]
						11 [20.00]	9 [20.00]	11 [20.00]	6 [20.00]	7 [20.00]	9 [20.00]
TSC6-33A						TSC6-32					

Names

0-2-3-5-7-9	0-3-5-6-8-10	0-2-5-7-8-10	0-2-4-7-9-10	0-2-4-6-9-11	0-1-3-5-7-10	0-2-4-5-7-9	0-3-5-7-8-10	0-2-5-7-9-10	0-2-4-7-9-11	0-1-3-5-8-10	0-2-3-5-7-10	
Raga Gauri Velavali	0-3-5-6-8-10 hexachord	Raga Navamano hari	0-2-4-7-9-10 hexachord	0-2-4-6-9-11 hexachord	Minor seventh (add 11, flat nine)	Diatonic hexachord Scottish hexatonic Arezzo major diatonic hexachord (the first sixth notes form the hexachord)	Phrygian hexatonic	Dominant thirteenth suspended fourth Thirteenth suspended fourth Mixolydian hexatonic P'yongjo (Korea)	Quintal hexatonic	Quartal hexatonic Ritsu (Japan)	Minor eleventh Minor hexatonic Hexatonic Minor Eskimo hexatonic 1 (Alaska (King Island))	
TSC6-33A												TSC6-32

Names

Appendix A continues on the next page.


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-2-4-6-8-11
34.45 %
0-1-3-5-7-9
34.12 %
0-3-4-6-8-10
33.67 %
0-2-5-6-8-10
31.89 %
0-2-4-7-8-10
33.00 %
0-2-4-6-9-10
34.56 %
0-2-4-6-7-9
37.35 %
0-3-5-7-9-10
37.80 %
0-2-5-7-9-11
38.58 %
0-1-3-6-8-10
37.13 %
0-2-3-5-8-10
34.46 %
0-2-4-5-7-10
35.13 %
TSC6-34A
TSC6-33B
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

TSC6-34A							TSC6-33B						
0-2-4-6-8-11	0-1-3-5-7-9	0-3-4-6-8-10	0-2-5-6-8-10	0-2-4-7-8-10	0-2-4-6-9-10	0-2-4-6-7-9	0-3-5-7-9-10	0-2-5-7-9-11	0-1-3-6-8-10	0-2-3-5-8-10	0-2-4-5-7-10		
6 [215.83]	7 [215.83]	10 [215.83]	0 [215.83]	2 [215.83]	4 [215.83]	4 [215.83]	7 [215.83]	9 [215.83]	10 [215.83]	0 [215.83]	2 [215.83]		
4 [165.83]	5 [165.83]	8 [165.83]	10 [165.83]	0 [165.83]	2 [165.83]	2 [182.50]	5 [182.50]	7 [182.50]	8 [182.50]	10 [182.50]	0 [182.50]		
0 [153.33]	1 [153.33]	4 [153.33]	6 [153.33]	8 [153.33]	10 [153.33]	7 [170.00]	10 [170.00]	0 [170.00]	1 [170.00]	3 [170.00]	5 [170.00]		
11 [150.00]	0 [150.00]	3 [150.00]	5 [150.00]	7 [150.00]	9 [150.00]	9 [162.50]	0 [162.50]	2 [162.50]	3 [162.50]	5 [162.50]	7 [162.50]		
8 [145.83]	9 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	6 [145.83]	6 [145.83]	9 [145.83]	11 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]		
2 [132.50]	3 [132.50]	6 [132.50]	8 [132.50]	10 [132.50]	0 [132.50]	0 [120.00]	3 [120.00]	5 [120.00]	6 [120.00]	8 [120.00]	10 [120.00]		
3 [83.33]	4 [83.33]	7 [83.33]	9 [83.33]	11 [83.33]	1 [83.33]	11 [95.83]	2 [95.83]	4 [95.83]	5 [95.83]	7 [95.83]	9 [95.83]		
9 [70.00]	10 [70.00]	1 [70.00]	3 [70.00]	5 [70.00]	7 [70.00]	1 [83.33]	4 [83.33]	6 [83.33]	7 [83.33]	9 [83.33]	11 [83.33]		
10 [65.83]	11 [65.83]	2 [65.83]	4 [65.83]	6 [65.83]	8 [65.83]	10 [53.33]	1 [53.33]	3 [53.33]	4 [53.33]	6 [53.33]	8 [53.33]		
1 [62.50]	2 [62.50]	5 [62.50]	7 [62.50]	9 [62.50]	11 [62.50]	8 [45.83]	11 [45.83]	1 [45.83]	2 [45.83]	4 [45.83]	6 [45.83]		
7 [50.00]	8 [50.00]	11 [50.00]	1 [50.00]	3 [50.00]	5 [50.00]	5 [20.00]	8 [20.00]	10 [20.00]	11 [20.00]	1 [20.00]	3 [20.00]		

to 220%												to 220%
to 210%												to 210%
to 200%												to 200%
to 190%												to 190%
to 180%												to 180%
to 170%												to 170%
to 160%												to 160%
to 150%												to 150%
to 140%												to 140%
to 130%												to 130%
to 120%												to 120%
to 110%												to 110%
to 100%												to 100%
to 90%												to 90%
to 80%	0-2-4-6-8-11 73.53 %	0-1-3-5-7-9 73.53 %	0-3-4-6-8-10 73.53 %	0-2-5-6-8-10 73.53 %	0-2-4-7-8-10 73.53 %	0-2-4-6-9-10 73.53 %						to 80%
to 70%												to 70%
to 60%												to 60%
to 50%							0-2-4-6-7-9 46.95 %	0-3-5-7-9-10 46.95 %	0-2-5-7-9-11 46.95 %	0-1-3-6-8-10 46.95 %	0-2-3-5-8-10 46.95 %	0-2-4-5-7-10 46.95 % to 50%
to 40%												to 40%
to 30%												to 30%
to 20%												to 20%
to 10%												to 10%
0% / 2 equal roots												0% / 2 equal roots
0% / 3 equal roots												0% / 3 equal roots
0% / 4+ equal roots												0% / 4+ equal roots
Root clarity	TSC6-34A						TSC6-33B					Root clarity

0-2-4-6-8-11	0-1-3-5-7-9	0-3-4-6-8-10	0-2-5-6-8-10	0-2-4-7-8-10	0-2-4-6-9-10	0-2-4-6-7-9	0-3-5-7-9-10	0-2-5-7-9-11	0-1-3-6-8-10	0-2-3-5-8-10	0-2-4-5-7-10
4 [575.56]	5 [575.56]	8 [575.56]	10 [575.56]	0 [575.56]	2 [575.56]	2 [575.56]	5 [575.56]	7 [575.56]	8 [575.56]	10 [575.56]	0 [575.56]
2 [331.67]	3 [331.67]	6 [331.67]	8 [331.67]	10 [331.67]	0 [331.67]	0 [391.67]	3 [391.67]	5 [391.67]	6 [391.67]	8 [391.67]	10 [391.67]
8 [230.00]	9 [230.00]	0 [230.00]	2 [230.00]	4 [230.00]	6 [230.00]	9 [255.00]	0 [255.00]	2 [255.00]	3 [255.00]	5 [255.00]	7 [255.00]
0 [218.75]	1 [218.75]	4 [218.75]	6 [218.75]	8 [218.75]	10 [218.75]	7 [243.75]	10 [243.75]	0 [243.75]	1 [243.75]	3 [243.75]	5 [243.75]
6 [198.75]	7 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]	4 [198.75]	4 [198.75]	7 [198.75]	9 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]
11 [175.00]	0 [175.00]	3 [175.00]	5 [175.00]	7 [175.00]	9 [175.00]	5 [143.75]	8 [143.75]	10 [143.75]	11 [143.75]	1 [143.75]	3 [143.75]
10 [98.75]	11 [98.75]	2 [98.75]	4 [98.75]	6 [98.75]	8 [98.75]	6 [140.00]	9 [140.00]	11 [140.00]	0 [140.00]	2 [140.00]	4 [140.00]
7 [97.22]	8 [97.22]	11 [97.22]	1 [97.22]	3 [97.22]	5 [97.22]	11 [81.67]	2 [81.67]	4 [81.67]	5 [81.67]	7 [81.67]	9 [81.67]
1 [81.67]	2 [81.67]	5 [81.67]	7 [81.67]	9 [81.67]	11 [81.67]	8 [62.22]	11 [62.22]	1 [62.22]	2 [62.22]	4 [62.22]	6 [62.22]
9 [72.92]	10 [72.92]	1 [72.92]	3 [72.92]	5 [72.92]	7 [72.92]	10 [53.47]	1 [53.47]	3 [53.47]	4 [53.47]	6 [53.47]	8 [53.47]
5 [50.00]	6 [50.00]	9 [50.00]	11 [50.00]	1 [50.00]	3 [50.00]	3 [33.33]	6 [33.33]	8 [33.33]	9 [33.33]	11 [33.33]	1 [33.33]
TSC6-34A									TSC6-33B		

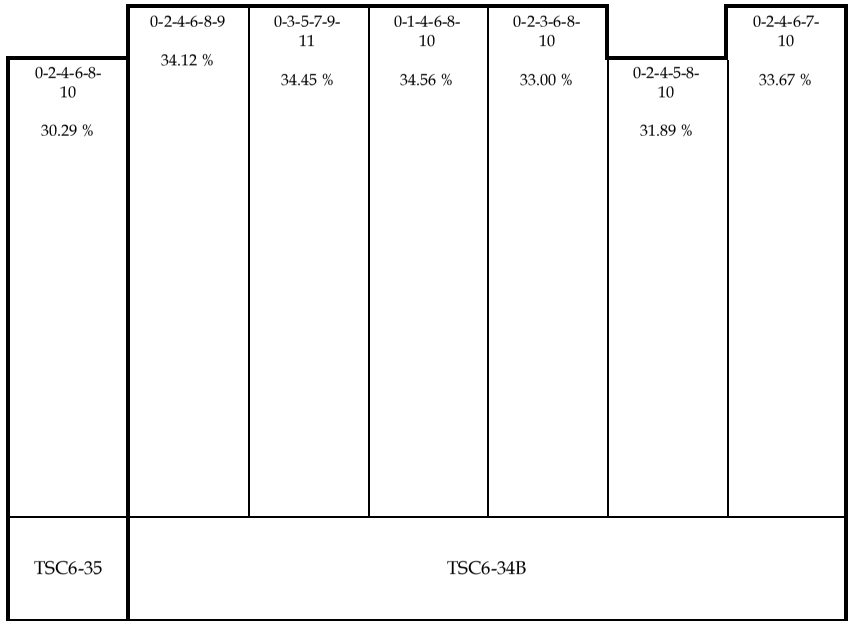
Names

0-2-4-6-8-11 Eskimo hexatonic	0-1-3-5-7-9 0-1-3-5-7-9 hexachord	0-3-4-6-8-10 0-3-4-6-8-10 hexachord	0-2-5-6-8-10 0-2-5-6-8-10 hexachord	0-2-4-7-8-10 0-2-4-7-8-10 hexachord	0-2-4-6-9-10 " Mystic chord"	TSC6-34A
0-2-4-6-7-9 Ancient Chinese	0-3-5-7-9-10 Pentatonic minor add 6	0-2-5-7-9-11 Lydian hexatonic	0-1-3-6-8-10 Raga Gurjari Todi scale	0-2-3-5-8-10 First inversion dominant eleventh	0-2-4-5-7-10 Dominant eleventh	TSC6-33B

Names

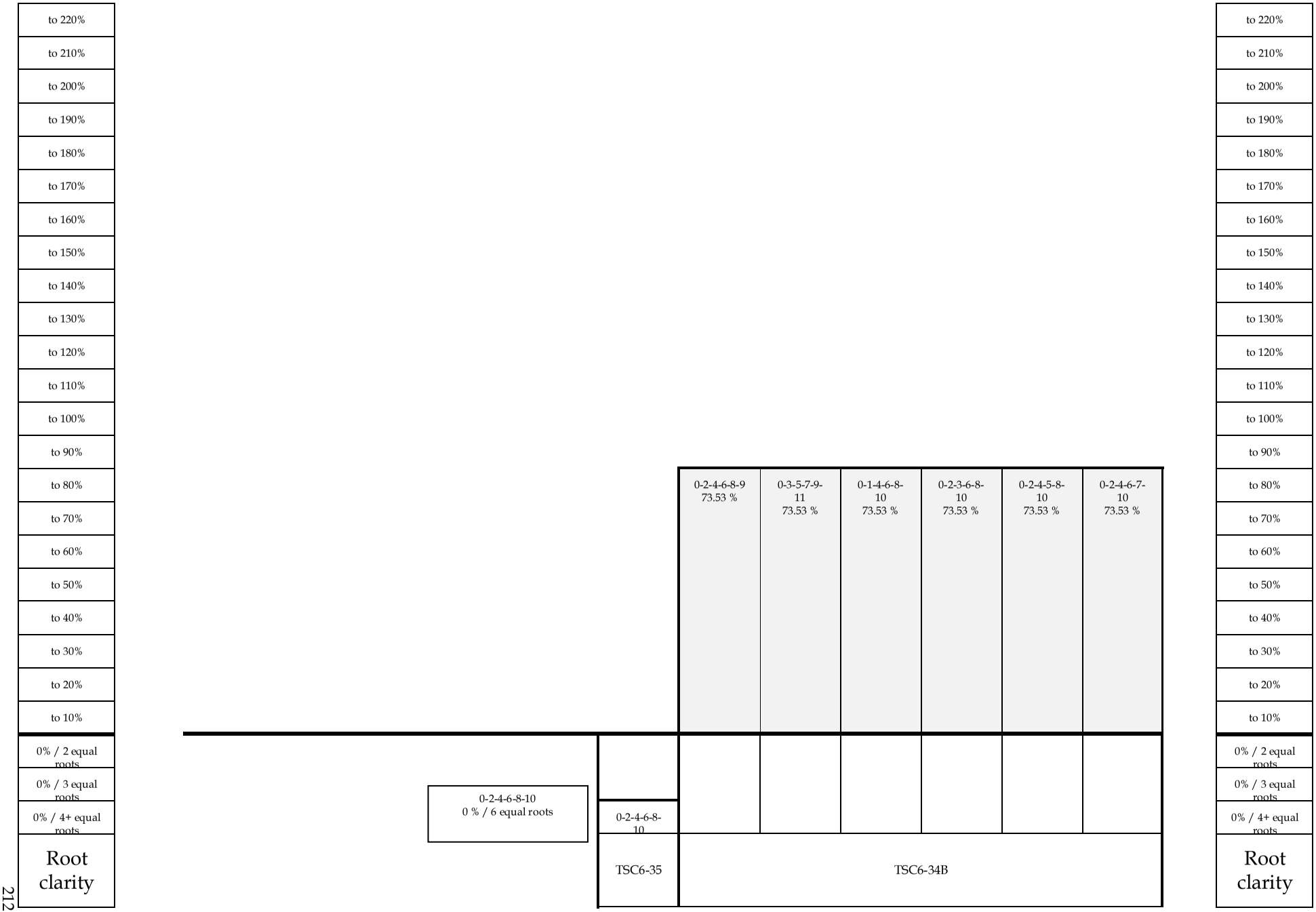
Appendix A continues on the next page.

to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series



to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series

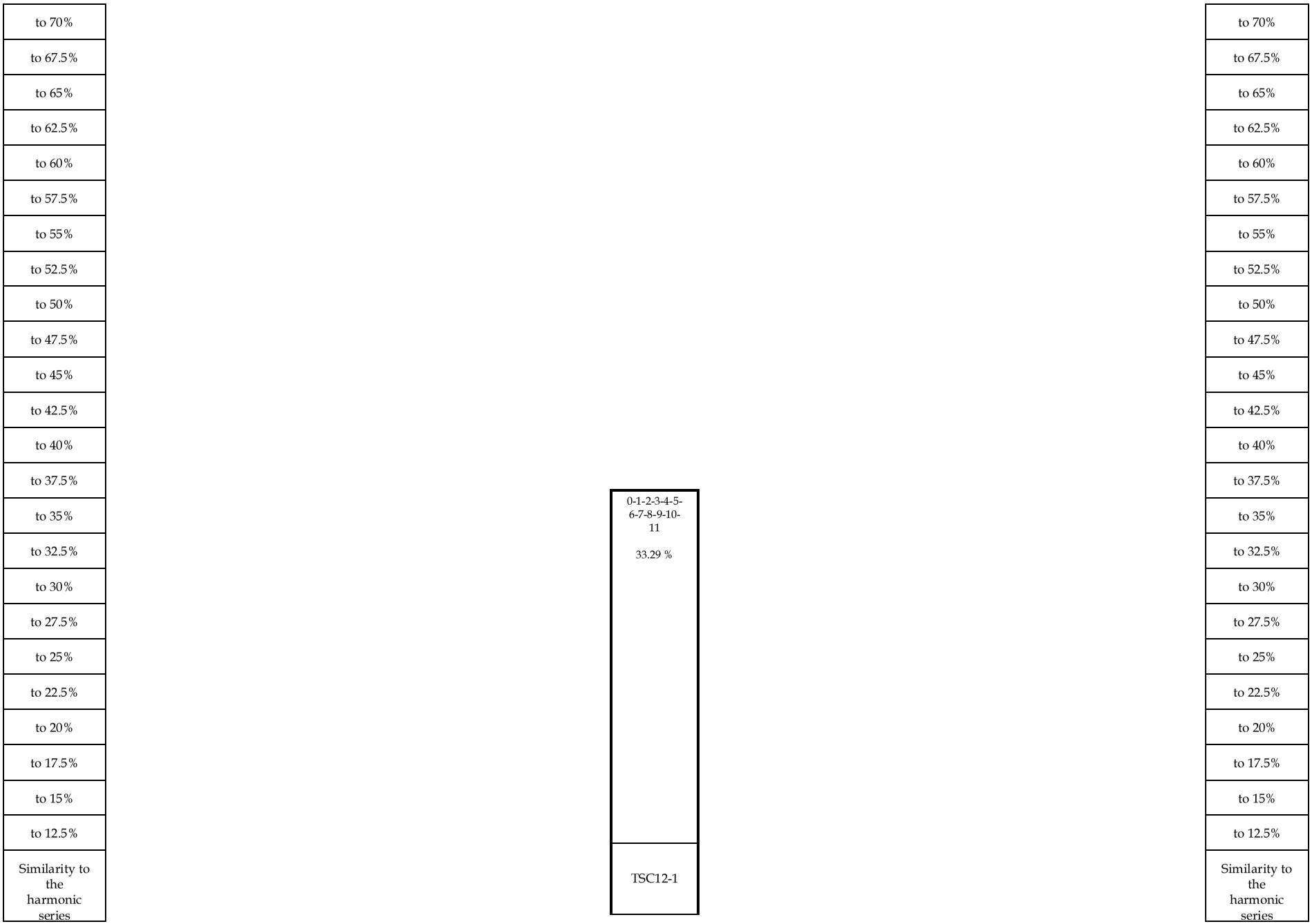
0-2-4-6-8-10	0-2-4-6-8-9	0-3-5-7-9-11	0-1-4-6-8-10	0-2-3-6-8-10	0-2-4-5-8-10	0-2-4-6-7-10
0 [165.83]	4 [215.83]	7 [215.83]	8 [215.83]	10 [215.83]	0 [215.83]	2 [215.83]
2 [165.83]	6 [165.83]	9 [165.83]	10 [165.83]	0 [165.83]	2 [165.83]	4 [165.83]
4 [165.83]	0 [153.33]	3 [153.33]	4 [153.33]	6 [153.33]	8 [153.33]	10 [153.33]
6 [165.83]	9 [150.00]	0 [150.00]	1 [150.00]	3 [150.00]	5 [150.00]	7 [150.00]
8 [165.83]	8 [145.83]	11 [145.83]	0 [145.83]	2 [145.83]	4 [145.83]	6 [145.83]
10 [165.83]	2 [132.50]	5 [132.50]	6 [132.50]	8 [132.50]	10 [132.50]	0 [132.50]
1 [50.00]	1 [83.33]	4 [83.33]	5 [83.33]	7 [83.33]	9 [83.33]	11 [83.33]
3 [50.00]	7 [70.00]	10 [70.00]	11 [70.00]	1 [70.00]	3 [70.00]	5 [70.00]
5 [50.00]	10 [65.83]	1 [65.83]	2 [65.83]	4 [65.83]	6 [65.83]	8 [65.83]
7 [50.00]	11 [62.50]	2 [62.50]	3 [62.50]	5 [62.50]	7 [62.50]	9 [62.50]
9 [50.00]	3 [50.00]	6 [50.00]	7 [50.00]	9 [50.00]	11 [50.00]	1 [50.00]
11 [50.00]						
TSC6-35	TSC6-34B					

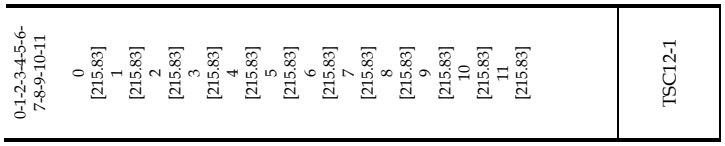

to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-2-4-6-8-9 73.53 %
0-3-5-7-9-11 73.53 %
0-1-4-6-8-10 73.53 %
0-2-3-6-8-10 73.53 %
0-2-4-5-8-10 73.53 %
0-2-4-6-7-10 73.53 %
0-2-4-6-8-10 0 % / 6 equal roots
0-2-4-6-8-10
TSC6-35
TSC6-34B

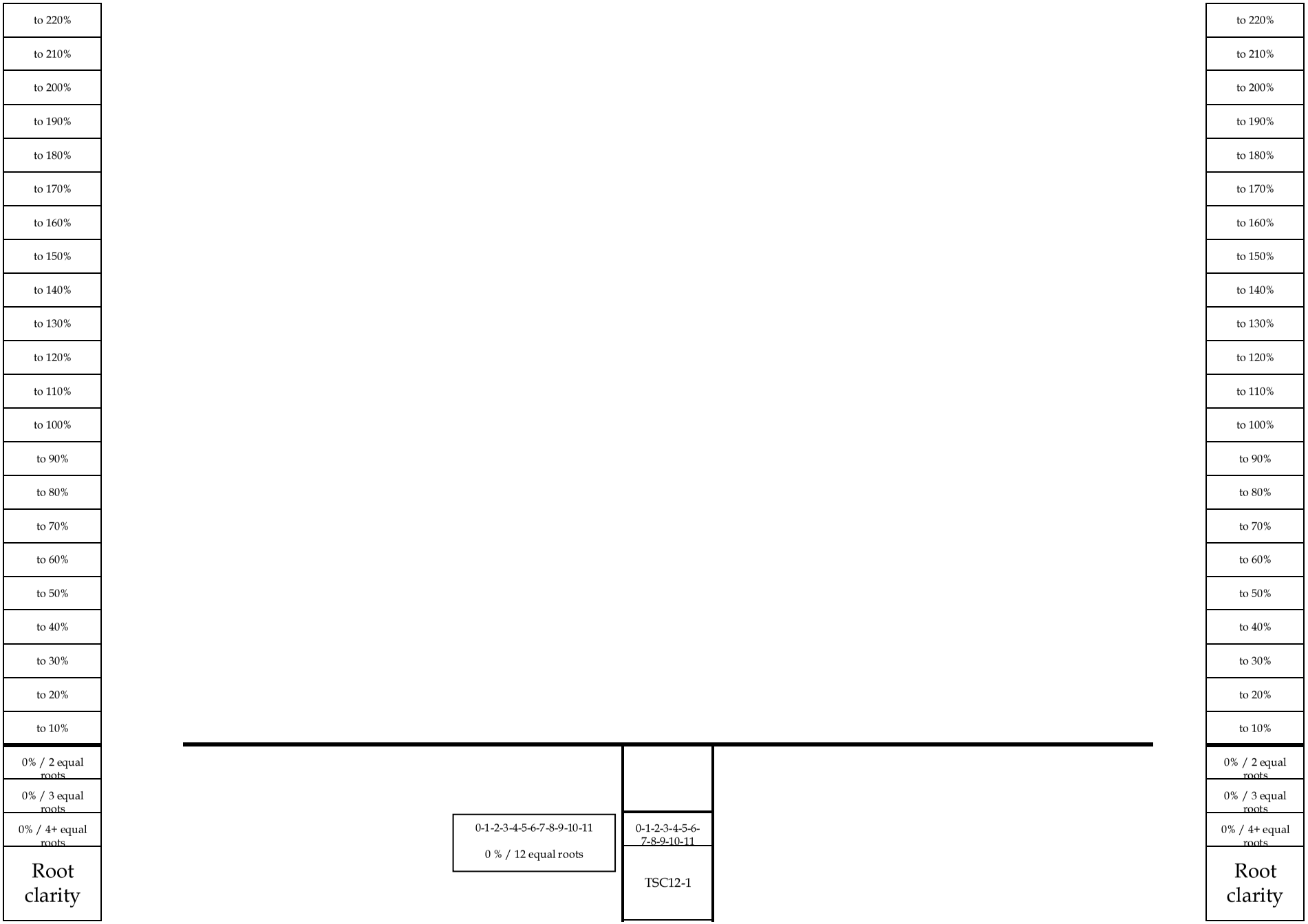
0-2-4-6-8-10	0-2-4-6-8-9	0-3-5-7-9-11	0-1-4-6-8-10	0-2-3-6-8-10	0-2-4-5-8-10	0-2-4-6-7-10
0 [331.67]	2 [575.56]	5 [575.56]	6 [575.56]	8 [575.56]	10 [575.56]	0 [575.56]
2 [331.67]	4 [331.67]	7 [331.67]	8 [331.67]	10 [331.67]	0 [331.67]	2 [331.67]
4 [331.67]	8 [230.00]	11 [230.00]	0 [230.00]	2 [230.00]	4 [230.00]	6 [230.00]
6 [331.67]	0 [218.75]	3 [218.75]	4 [218.75]	6 [218.75]	8 [218.75]	10 [218.75]
8 [331.67]	6 [198.75]	9 [198.75]	10 [198.75]	0 [198.75]	2 [198.75]	4 [198.75]
10 [331.67]	9 [175.00]	0 [175.00]	1 [175.00]	3 [175.00]	5 [175.00]	7 [175.00]
1 [50.00]	10 [98.75]	1 [98.75]	2 [98.75]	4 [98.75]	6 [98.75]	8 [98.75]
3 [50.00]	5 [97.22]	8 [97.22]	9 [97.22]	11 [97.22]	1 [97.22]	3 [97.22]
5 [50.00]	11 [81.67]	2 [81.67]	3 [81.67]	5 [81.67]	7 [81.67]	9 [81.67]
7 [50.00]	7 [72.92]	10 [72.92]	11 [72.92]	1 [72.92]	3 [72.92]	5 [72.92]
9 [50.00]	1 [50.00]	4 [50.00]	5 [50.00]	7 [50.00]	9 [50.00]	11 [50.00]
11 [50.00]						
TSC6-35	TSC6-34B					

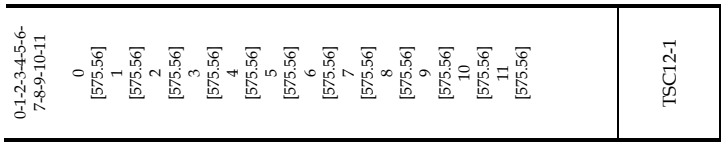
0-2-4-6-8-10 Whole-Tone Collection I Whole-tone I Whole-tone scale Anhemitonic hexatonic Hexatonic Anhemitonic Messiaen mode 1 Messiaen Modus 1	0-2-4-6-8-9 0-2-4-6-8-9 hexachord	0-3-5-7-9-11 0-3-5-7-9-11 hexachord	0-1-4-6-8-10 Seven flat ninth sharp eleventh augmented	0-2-3-6-8-10 Takemitsu Tree Line mode 2	0-2-4-5-8-10 0-2-4-5-8-10 hexachord	0-2-4-6-7-10 Augmented eleventh
TSC6-35	TSC6-34B					

Twelve-pitch chords
(chromatic scales as chords)


to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series
0-1-2-3-4-5-6-7-8-9-10-11
33.29 %
TSC12-1
to 70%
to 67.5%
to 65%
to 62.5%
to 60%
to 57.5%
to 55%
to 52.5%
to 50%
to 47.5%
to 45%
to 42.5%
to 40%
to 37.5%
to 35%
to 32.5%
to 30%
to 27.5%
to 25%
to 22.5%
to 20%
to 17.5%
to 15%
to 12.5%
Similarity to the harmonic series


0-1-2-3-4-5-6-7-8-9-10-11
0
[215.83]
1
[215.83]
2
[215.83]
3
[215.83]
4
[215.83]
5
[215.83]
6
[215.83]
7
[215.83]
8
[215.83]
9
[215.83]
10
[215.83]
11
[215.83]
TSC12-1


to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity
0-1-2-3-4-5-6-7-8-9-10-11
0 % / 12 equal roots
0-1-2-3-4-5-6-7-8-9-10-11
TSC12-1
to 220%
to 210%
to 200%
to 190%
to 180%
to 170%
to 160%
to 150%
to 140%
to 130%
to 120%
to 110%
to 100%
to 90%
to 80%
to 70%
to 60%
to 50%
to 40%
to 30%
to 20%
to 10%
0% / 2 equal roots
0% / 3 equal roots
0% / 4+ equal roots
Root clarity



Root candidates and weights

0-1-2-3-4-5-6-7-8-9-10-11 Chromatic scales as chords	TSC12-1

Appendix B

How harmonic similarity (hsim) values were calculated

The harmonic similarity measure for each chord was calculated as the sum of the harmonic similarity value for each just interval in the chord, divided by the divisor for its cardinality.

Intervals	Just interval size (cents)	Hsim
0-1	119.44	13.33
0-2	203.91	22.22
0-3	315.64	33.33
0-4	386.31	40.00
0-5	498.04	50.00
0-6	582.51	31.43
0-7	701.96	66.67
0-8	813.69	30.00
0-9	884.36	46.67
0-10	1017.60	28.89
0-11	1088.27	18.33

Cardinality	Divisor
1	1
2	1
3	3
4	6
5	10
6	15
7	21
8	28
9	36
10	45
11	55

Example:

The harmonic similarity measure for 0-4-7 is:

0-4 = 40.00

0-7 = 66.67

4-7 = 0-3 = 33.33

(40.00 + 66.67 + 33.33) / 3 = **46.67**

Source:

Gill KZ, Purves D (2009) *A Biological Rationale for Musical Scales*. PLoS ONE 4(12): e8144. doi:10.1371/journal.pone.0008144

Footnotes

1. Tymoczko D (2011) at p. 28.
2. An example of this classification and the reasons for using it can be found in The Table of Pitch Class Sets and the explanation below it on Larry J. Solomon's website at: http://solomonsmusic.net/pcsets.htm.
3. Bowling DL, and Purves D (2015).
4. It is not the intent of this book to explore the complex fields of acoustics and psychoacoustics.
5. Gill KZ, and Purves D (2009) at p. 1.
6. Gill KZ, and Purves D (2009) at p. 6, cross-references omitted.
7. Ross D, Choi J, and Purves D (2007) at p. 9854.
8. Gill KZ, and Purves D (2009) at p. 1.
9. Gill KZ, and Purves D (2009) at p. 2, cross-references omitted.
10. Gill KZ, and Purves D (2009) at p. 2, cross-references omitted. This is the explanation provided by Gill and Purves for their choice of frequency ratios and percentage similarity: "Because musical scale tones are not always defined by a single frequency ratio (e.g., the ratios of 7:5 or 10:7 can both represent a tritone), the algorithm we used allowed tones within a specific frequency distance to represent the same scale tone. To our knowledge, there is no psychoacoustical data on the size of the frequency window within which intervals are considered musically equivalent. We thus defined the window based on musical practice. Twenty-two cents was used because it is the maximum frequency distance between scale tones that are considered musically equivalent in Western music (i.e., the interval between the minor sevenths defined by ratios of 9:5 and 16:9); it is also the minimum frequency distance between two tones that are considered unique in classical Indian music. Note that 22 cents is significantly larger than the just noticeable frequency difference between tones (around five cents), implying that the size of the window is not based on the resolution of the auditory system. If two or more ratios fell within the 22 cent window, the algorithm defaulted to the ratio yielding the highest percentage similarity from any comparison. For example, if 9:8 or 10:9 represented the second scale degree of a scale being tested (these two intervals are within 22 cents of each other), the algorithm would use 9:8 rather than 10:9 to form an interval with a perfect fifth (3:2) because this choice produces the interval (4:3 versus 27:20) with the higher percentage similarity. Conversely, the algorithm would use 10:9 rather than 9:8 to form an interval with a major sixth (5:3) because this choice produces the interval (3:2 versus 40:27) with the higher percentage similarity."
11. Blatter A (2007) at p. 268.
12. Gill KZ, and Purves D (2009) at p. 3, cross-references omitted.

13. These percentage similarity values are derived from Table 1 of Gill KZ, and Purves D (2009) at p. 5.
14. Terhardt E (2000d).
15. Terhardt E (2000c).
16. Terhardt E, Stoll G, and Seewann M (1982a) at p. 679.
17. Terhardt E (2000b).
18. Terhardt E, Stoll G, and Seewann M (1982b) at p. 677. The results of the analysis by the authors of the A major and minor triads are difficult to interpret clearly. The article does not indicate whether the piano was in good condition and had been properly tuned prior to the tests, what acoustic environment the piano was played in, and whether each chordal pitch was played with identical dynamics.
19. Brandt H (2009) at p. 4.
20. Gill KZ, and Purves D (2009) at p. 7.
21. Within the generic timbre, there are several subsets of timbres, depending on which harmonics and subharmonics are predominant.

Harmonics and subharmonics	**Subset 1**	**Subset 2**	**Subset 3**	**Subset 4**	**Subset 5**	**Subset 6**	**Subset 7**	**Subset 8**
	Percent	**Percent**	**Percent**	**Percent**	**Percent**	**Percent**	**Percent**	**Percent**
The 9th harmonic	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50
The 8th harmonic	<100.00	<100.00	<100.00	<100.00	<100.00	<100.00	**100.00**	**100.00**
The 7th harmonic	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00
The 6th harmonic	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**
The 5th harmonic	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3
The 4th harmonic	<100.00	<100.00	<100.00	<100.00	**100.00**	**100.00**	<100.00	<100.00
The 3rd harmonic	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00
The 2nd harmonic	<100.00	<100.00	**100.00**	**100.00**	<100.00	<100.00	<100.00	<100.00
The note played	**100.00**	**100.00**	<100.00	<100.00	<100.00	<100.00	<100.00	<100.00
The note played	**100.00**	**100.00**	<100.00	<100.00	<100.00	<100.00	<100.00	<100.00
The 2nd subharmonic	<100.00	<100.00	**100.00**	**100.00**	<100.00	<100.00	<100.00	<100.00
The 3rd subharmonic	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00
The 4th subharmonic	<100.00	<100.00	<100.00	<100.00	**100.00**	**100.00**	<100.00	<100.00
The 5th subharmonic	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3	33 1/3
The 6th harmonic	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**	<50.00	**50.00**
The 7th subharmonic	20.00	20.00	20.00	20.00	20.00	20.00	20.00	20.00
The 8th subharmonic	<100.00	<100.00	<100.00	<100.00	<100.00	<100.00	**100.00**	**100.00**
The 9th subharmonic	12.50	12.50	12.50	12.50	12.50	12.50	12.50	12.50

22. The values derived with the generic timbre also influence the reasoning and the mathematics behind the selection of the specific subharmonic enhancement percentage in subchapter 1.2.5 (The subharmonic enhancement multiplier).
23. My subjective listening tests consisted of performing chords on an electric piano in close position in the fourth octave with their root candidates being tested one, two and three octaves below, but not lower than F1, the lowest possible virtual root. Note that the relative subjective weight of root candidates can vary to some extent in different octaves.
24. Terhardt E, Stoll G, and Seewann M (1982a). The following is an excerpt from Terhardt's complete psychoacoustic model: Terhardt E (2004) *ptp2svp Conversion of*

part-tone patterns into spectral-virtual-pitch patterns, accessed via www.uni-graz.at/richard.parncutt/ptp2svpCode.html in November 2006:

```
/*Evaluation of subharmonic coincidences; creation of the virtual-pitch
    pattern.*/

int signum(int n) {
if(n==0) return(0);
if(n>0) return(1);
return(-1);
}

double truvp(double nomvp, int i, int m) {
double x;
if(nomvp > 0.5) return(0.0);          /* for >500Hz true v. pitch is undefined
    */
x =1.0E-3*(18.0+2.5*(double)m -(50.0-7.0*(double)m)*nomvp +
    0.1/nomvp/nomvp);
x= -x*(double)signum(m-1);
x=x+1.0+spp.shift[i];
return(nomvp*x);
}
```

I cannot say whether the results provided, when using this polygon model, are more or less accurate than the results produced by Terhardt's complete psychoacoustic model. However, the subharmonic enhancement multiplier in our method, when used together with the timbre-weighted algorithm in Figure 1.13, provides more realistic results than Terhardt's simplified algorithm in Figure 1.6.

25. Terhardt E, Stoll G, and Seewann M (1982b) at p. 677.
26. Bowling DL, and Purves D (2015).
27. Tymoczko D (2011) at p. 4. Tymoczko also discusses other elements of tonality that are not addressed here.

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Terhardt E (2000c) *Spectral pitch*. www.mmk.e-technik-tu-muenchen.de/persons/ter/top/specpitch.html (accessed in October 2006).

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The Sound of Chords, by Harry Laurence Hart, is a unique and practical method created for young composers for the purpose of accurately evaluating the sound of any chord, and for comparing the sounds of different chords. This book will also help them understand relationships between chords that contain different numbers of pitches, and help them develop chord progressions.